MINDSET MAKERS

UPSC
IAS / IFoS
Mathematics
Optional

Mindset Makers
U.P.S.C. I.I.T.
Prepare in Right Way

Previous Years
Paper-2 [up to year 2022]

Chapterwise Questions Collection

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GROUP THEORY

1. GROUPS AND SUBGROUPS
2. CYCLIC GROUPS
3. COSETS, NORMAL SUBGROUPS & QUOTIENT GROUPS
4. HOMOMORPHISM AND AUTOMORPHISMS
5. PERMUTATION GROUPS

RING THEORY

1. RINGS AND FIELDS
2. IDEALS AND QUOTIENT RINGS
3. HOMOMORPHISM OF RINGS
4. EUCLIDEAN RINGS, PID
5. POLYNOMIAL RINGS, UFD

CHAPTER 1. GROUPS AND SUBGROUPS

NUMBER THEORY

Q1(a) Let G be a group of order 10 and G' be a group of order 6. Examine whether there exists a homomorphism of G onto G'. **UPSC CSE 2023**

Q1. Let $m_1, m_2, ..., m_k$ be positive integers and $d > 0$ the greatest common divisor of $m_1, m_2, ..., m_k$. Show that there exist integers $x_1, x_2, ..., x_k$ such that $d = x_1m_1 + x_2m_2 + ... + x_km_k$. **[1a UPSC CSE 2021]**

Q2. Show that there are infinitely many subgroups of the additive group $\mathbf{Q}$ of rational numbers.
[4a UPSC CSE 2021]

Q1. Let p be a prime number. Then show that
$(p-1)1+1 \equiv \text{mod}(p)$
Also, find the remainder when $6^{44} \cdot (22)1+3$ is divided by 23. **[1a 2020 IFoS]**

Q2. If in the group G, $a^5 = e, aba^{-1} = b^2$ for some $a, b \in G$, find the order of *b*.
[3a 2019 IFoS]

Q3. Prove that every group of order four is Abelian. **[1a 2017 IFoS]**

Q4. Let G be the set of all real numbers except -1 and define $a * b = a + b + ab$ $\forall a, b \in G$. Examine if G is an Abelian group under *. **[2a 2017 IFoS]**

Q5. Prove that the set of all bijective functions from a non-empty set X onto itself is a group with respect to usual composition of functions. **[1a 2016 IFoS]**

Q6. If G is a group in which $(a \cdot b)^4 = a^4 \cdot b^4, (a \cdot b)^5 = a^5 \cdot b^5$ and $(a \cdot b)^6 = a^6 \cdot b^6$ for all $a, b \in G$, then prove that G is Abelian. **[1a 2014 IFoS]**

Q7. Give an example of an infinite group in which every element has finite order.
[1b UPSC CSE 2013]

Q8. Prove that if every element of a group $(G, 0)$ be its own inverse, then it is an abelian group.
[1b 2013 IFoS]

Q9. How many elements of order 2 are there in the group of order 16 generated by *a* and *b* such that the order of *a* is 8, the order of *b* is 2 and $bab^{-1} = a^{-1}$. **[1a UPSC CSE 2012]**

Q10. Show that the set
$G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by

$$f_1(z) = z,\ f_2(z) = 1 - z$$

$$f_3(z) = \frac{z}{(z-1)},\ f_4(z) = \frac{1}{z}$$

$f_5(z) = \frac{1}{(1-z)}$ and $f_6(z) = \frac{(z-1)}{z}$ is a non-abelian group of order 6 w.r.t composition of mappings. **[1a UPSC CSE 2011]**

Q11. Let *a* and *b* be elements of a group with $a^2 = e$, $b^6 = e$ and $ab = b^4 a$. Find the order of ab, and express its inverse in each of the forms $a^m b^n$ and $b^m a^n$. **[4a UPSC CSE 2011]**

Q12. Let G be a group, and *x* and *y* be any two elements of G. IF $y^5 = e$ and $yxy^{-1} = x^2$, then show that $O(x) = 31$, where *e* is the identity element of G and $x \neq e$. **[1a 2011 IFoS]**

Q13. Let $G = \mathbf{R} - \{-1\}$ be the set of all real numbers omitting -1. Define the binary relation * on G by $a * b = a + b + ab$. Show $(G, *)$ is a group and it is abelian. **[1a UPSC CSE 2010]**

Q14. Let
$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in \mathbf{R}, a \neq 0 \right\}$. Show that G is group under matrix multiplication.

[1a 2010 IFoS]

Q15. Show that zero and unity are only idempotent of Z_n if $n = p^r$, where *p* is a prime.

[3a 2010 IFoS]

CHAPTER 2. CYCLIC GROUPS

Q1. Let G be a finite cyclic group of order *n*. Then prove that G has $\phi(n)$ generators (where ϕ is Euler's ϕ-function). **[2a UPSC CSE 2020]**

Q2. Let G be a finite group and let *p* be a prime. If p^m divides order of G, then show that G has a subgroup of order p^m, where *m* is a positive integer. **[3b 2020 IFoS]**

Q3. Let p be a prime number and $\mathbf{Z}_p$ denote the additive group of integers modulo p. Show that every non-zero elements of $\mathbf{Z}_p$ generates $\mathbf{Z}_p$. **[2b UPSC CSE 2016]**

Q4. Let G be a group of order pq, where p and q are prime numbers such that $p > q$ and $q \times (p-1)$. Then prove that G is cyclic. **[2b 2016 IFoS]**

Q5. How many generators are there of the cyclic group G of order 8? Explain. Taking a group $\{e, a, b, c\}$ of order 4, where *e* is the identity, construct composition tables showing that one is cyclic while the other is not. **[1a UPSC CSE 2015]**

Q6. If in a group G there is an element *a* of order 360, what is the order of a^{220}? Show that if G is a cyclic group of order *n* and *m* divides *n*, then G has a subgroup of order *m*. **[1a 2015 IFoS]**

Q7. Prove that a group of prime order is abelian. How many generators are there of the cyclic group $(G,\cdot)$ of order 8? **[1e UPSC CSE 2011]**

Q8. Given an example of group G in which every proper subgroup is cyclic but the group itself is not cyclic. **[2a UPSC CSE 2011]**

Q9. Let G be a group of order $2p, p$ prime. Show that either G is cyclic or G is generated by $\{a,b\}$ with relations $a^p = e = b^2$ and $bab = a^{-1}$. **[3b 2011 IFoS]**

Q10. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify. **[1b UPSC CSE 2010]**

Q11. Determine the number of homeomorphisms from the additive group $\mathbf{Z}_{15}$ to the additive group $\mathbf{Z}_{10}$.($\mathbf{Z}_n$ is the cyclic group of order *n*). **[1b UPSC CSE 2009]**

CHAPTER 3. COSETS, NORMAL SUBGROUPS & QUOTIENT GROUPS
UPDATED

Q1. Let P be a Sylow *p*-subgroup of a group G and H is any *p*-subgroup of G such that $HP = PH$. Then show that $H \subseteq P$.
(ii) Show that every group of order 15 is cyclic. **[3b IFoS 2022]**

Q1. Let G be a finite group, H and K subgroups of G such that $K \subset H$. Show that $(G:K)=(G:H)(H:K)$. **[1a UPSC CSE 2019]**

Q2. Write down all quotient groups of the group Z_{12}. **[2b UPSC CSE 2019]**

Q3. Prove that a non-commutative group of order $2n$, where n is an odd prime must have a subgroup of order n. **[1a 2018 IFoS]**

Q4. Let H be a cyclic subgroup of a group G. If H be a normal subgroup of G, prove that every subgroup of H is a normal subgroup of G. **[4c 2018 IFoS]**

Q5. Let H and K are finite normal subgroups of co-prime order of a group G. Prove that $hk = kh \forall h \in H$ and $k \in K$. **[2b 2017 IFoS]**

Q6. Let G be the set of all real 2×2 matrices $\begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$, where $xz \neq 0$. Show that G is a group under matrix multiplication. Let N denote the subset $\left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in \mathbf{R} \right\}$. Is N a normal subgroup of G? Justify your answer. **[1a UPSC CSE 2014]**
Q7. Prove that a non-empty subset H of a group G is normal subgroup of $G \Leftrightarrow$ for all $x, y \in H, g \in G, (gx)(gy)^{-1} \in H$. **[1a 2009 IFoS]**
Q8. If G is a finite Abelian group, then show that $O(a,b)$ is a divisor of 1 c.m. of $O(a), O(b)$.**[1d 2009 IFoS]**

CHAPTER 4. HOMOMORPHISM AND AUTOMORPHISMS

UPDATED
Q1. Show that the multiplicative group $G = \{1, -1, i, -i\}$, where $i = \sqrt{(-1)}$, is isomorphic to the group $FG' = (\{0,1,2,3\}, +_4)$. **[1a UPSC CSE 2022]**
Q2. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.
[2b UPSC CSE 2022]

Q3. Let G be a finite commutative group. Let $n \in \mathbf{Z}$ be such that *n* and order of G are relatively prime. Show that the function $\phi : G \to G$ defined by $\phi(a) = a^n$, for all $a \in G$, is an isomorphism of G onto G. **[1a IFoS 2021]**

Q1. If G and H are finite groups whose orders are relatively prime, then prove that there is only one homomorphism from G to H, the trivial one. **[2a UPSC CSE 2019]**
Q2. Show that the quotient group of $(\mathbf{R}, +)$ modulo $\mathbf{Z}$ is isomorphic to the multiplicative group of complex numbers on the unit circle in the complex plane. Here $\mathbf{R}$ is the set of real numbers and $\mathbf{Z}$ is the set of integers. **[2a UPSC CSE 2018]**
Q3. Find all the homeomorphisms from the group $(\mathbf{Z}, +)$ to $(\mathbf{Z}_4, +)$. **[2a 2018 IFoS]**
Q4. Show that the groups $\mathbf{Z}_5 \times \mathbf{Z}_7$ and $\mathbf{Z}_{35}$ are isomorphic. **[3a UPSC CSE 2017]**
Q5. Let G be the group of non-zero complex numbers under multiplication, and let N be the set of complex numbers of absolute value 1. Show that G/N is

isomorphic to the group of all positive real numbers under multiplication. **[2a 2011 IFoS]**

Q6. Let $(\mathbf{R}^*,\cdot)$ be the multiplicative group of non-zero reals and $(GL(n,\mathbf{R})X)$ be the multiplicative group of $n\times n$ non-singular real matrices. Show that the quotient group $GL(n,\mathbf{R})/SL(n,\mathbf{R})$ and $(\mathbf{R}^*,\cdot)$ are isomorphic where $SL(n,\mathbf{R})=\{A\in GL(n,\mathbf{R})/\det A=1\}$. What is the centre of GL $(n,\mathbf{R})$? **[2a UPSC CSE 2010]**

Q7. Prove or disprove that $(\mathbf{R},+)$ and $(\mathbf{R}^+,\cdot)$ are isomorphic groups where $\mathbf{R}^+$ denotes the set of all positive real numbers. **[2b 2010 IFoS]**

Q8. If $\mathbf{R}$ is the set of real numbers and $\mathbf{R}_+$ is the set of positive real numbers, show that $\mathbf{R}$ under addition $(\mathbf{R},+)$ and $\mathbf{R}_+$ under multiplication $(\mathbf{R}_+,\cdot)$ are isomorphic. Similarly if $\mathbf{Q}$ is the set of rational numbers and $\mathbf{Q}_+$ the set of positive rational numbers are $(\mathbf{Q},+)$ and $(\mathbf{Q}_+,\cdot)$ isomorphic? Justify your answer. **[1a UPSC CSE 2009**

CHAPTER 5. PERMUTATION GROUPS

UPDATED

Q1. Find all the Sylow *p*-subgroups of S_4 and show that none of them is normal. **[2a IFoS 2022]**

Q1. Let S_3 and Z_3 be permutation group on 3 symbols and group of residue classes module 3 respectively. Show that there is no homomorphism of S_3 in Z_3 except the trivial homomorphism.

[1a UPSC CSE 2020]

Q2. Show that the smallest subgroup V of A_4 containing $(1,2)(3,4)$, $(1,3)(2,4)$ and $(1,4)(2,3)$ is isomorphic to the Klein 4-group. **[4a 2019 IFoS]**

Q3. Let G be a group of order *n*. Show that G is isomorphic to a subgroup of the permutation group S_n. **[1b UPSC CSE 2017]**

Q4. Show that any non-abelian group of order 6 is isomorphic to the symmetric group S_3.

[2a 2016 IFoS]

Q5. What is the maximum possible order of a permutation in S_8, the group of permutations on the eight numbers $\{1,2,3,...,8\}$? Justify your answer. (Majority of marks will be given for the justification.) **[3a 2015 IFoS]**

Q6. What are the orders of the following permutations in S_{10}?

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$ and $(1\ 2\ 3\ 4\ 5)(6\ 7)$. **[2a UPSC CSE 2013]**

Q7. What is the maximal possible order of an element in S_{10}? Why? Give an example of such an element. How many elements will there be in S_{10} of that order? **[2b UPSC CSE 2013]**

Q8. How many conjugacy classes does the permutation group S_5 of permutations 5 numbers have? Write down one element in each class (preferably in terms of cycles). **[2a UPSC CSE 2012]**

Q9. Show that in a symmetric group S_3, there are four elements σ satisfying $\sigma^2 =$Identity and three elements satisfying $\sigma^3 =$Identity. **[2a 2012 IFoS]**

Q10. Show that the alternating group on four letters A_4 has no subgroup of order 6.

[2b UPSC CSE]

CHAPTER 1. RINGS AND FIELDS

Q1. Let K be a finite field. Show that the number of elements in K is p^n, where p is a prime, which is characteristic of K and $n \geq 1$ is an integer. Also, prove that $\frac{Z_3[x]}{(X^2+1)}$ is a field. How many elements does this field have? **[4b 2020 IFoS]**

Q2. Let R be an integral domain. Then prove that ch R (characteristic of R) is 0 or a prime.

[1a 2019 IFoS]

Q3. Find all the proper subgroups of the multiplicative group of the field $(\mathbf{Z}_{13}, +_{13}, \times_{13})$, where $+_{13}$ and $\times_{13}$ represent addition modulo 13 and multiplication modulo 13 respectively.

[3a UPSC CSE 2018]

Q4. Give an example of a ring having identity but a subring of this having a different identity.

[1b UPSC CSE 2015]

Q5. Do the following sets form integral domains with respect to ordinary addition and multiplication? If so, state if they are fields:

(i) The set of numbers of the form $b\sqrt{2}$ with b rational

(ii) The set of even integers

(iii) The set of positive integers. **[4a UPSC CSE 2015]**

Q6. If p is a prime number and e a positive integer, what are the elements 'a' in the ring $\mathbf{Z}_{p^e}$ of integers modulo p^e such that $a^2 = a$? Hence (or otherwise) determine the elements in $\mathbf{Z}_{35}$ such that $a^2 = a$. **[2a 2015 IFoS]**

Q7. Show that $\mathbf{Z}_7$ is a field. Then find $\left(|5|+|6|\right)^{-1}$ and $\left(-|4|\right)^{-1}$ in $\mathbf{Z}_7$. **[2a UPSC CSE 2014]**

Q8. Show that the set $\left\{a+b\omega : \omega^3 = 1\right\}$, where a and b are real numbers, is a field with respect to usual addition and multiplication. **[3a UPSC CSE 2014]**

Q9. Prove that the set $\mathbf{Q}\left(\sqrt{5}\right) = \left\{a + b\sqrt{5} : a, b \in \mathbf{Q}\right\}$ is a commutative ring with identity.

[4a UPSC CSE 2014]

Q10. Let J_n be the set of integers mod n. Then prove that J_n is a ring under the operations of addition and multiplication mod n. Under what conditions on n, J_n is a field? Justify your answer.

[2a IFoS 2014]

Q11. Show that any finite integral domain is a field. **[2a 2013 IFoS]**

Q12. Every field is an integral domain - Prove it. **[2b 2013 IFoS]**

Q13. Show that every field is without zero divisor. **[1b 2012 IFoS]**

Q14. Let Q be the set of all rotational numbers. Show that $Q\left(\sqrt{2}\right) = \left\{a + b\sqrt{2} : a, b \in Q\right\}$ is a field under the usual addition and multiplication. **[1b 2011 IFoS]**

Q15. Let $C = \left\{f : I = [0,1] \to \mathbf{R} \mid f \text{ if continuous}\right\}$. Show C is a commutative ring with 1 under point wise addition and multiplication. Determine whether C is an integral domain. Explain.

[2b UPSC CSE 2010]

Q16. Let F be a field of order 32. Show that the only subfields of F are F itself and $\{0,1\}$.

[1b 2010 IFoS]

Q17. Show that a field is an integral domain and a non-zero finite integral domain is a field.

[4b 2009 IFoS]

Q18. Find the multiplicative inverse of the element

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

of the ring, M, of all matrices of order two over the integers. **[2c 2009 IFoS**

CHAPTER 2. IDEALS AND QUOTIENT RINGS

Q3. (a) Prove that x^2+1 is an irreducible polynomial in $Z_3[x]$. Further show that the quotient ring $\dfrac{Z_3[x]}{<x^2+1>}$ is a field of 9 elements. **UPSC CSE 2023** **(15)**

Q1. Let R be a principal ideal domain. Show that every ideal of a quotient ring of R is principal ideal and R/P is a principal ideal domain for a prime ideal P of R. **[1b UPSC CSE 2020]**

Q2. Let R be a non-zero commutative ring with unity. Show that M is a maximal ideal in a ring R if and only if R/M is a field. **[2a 2020 IFoS]**

Q3. Show by an example that in a finite commutative ring, every maximal ideal need not be prime.

[3b 2018 IFoS]

Q4. Let A be an ideal of a commutative ring R and $B=\{x\in R : x^n \in A \text{ for some positive integer } n\}$. Is B an ideal of R? Justify your answer.

[2c 2017 IFoS]

Q5. Let $R^C=$ ring of all real valued continuous functions on $[0,1]$, under the operations

$(f+g)x=f(x)+g(x)$

$(fg)x=f(x)g(x)$.

Let $M=\left\{f\in R^C \,\middle|\, f\left(\frac{1}{2}\right)=0\right\}$. Is M a maximal ideal of R? Justify your answer.

[3b UPSC CSE 2013]

Q6. Prove that:

(i) the intersection of two ideals is an ideal

(ii) a field has no proper ideals. **[3b 2013 IFoS]**

Q7. Is the ideal generated by 2 and X in the polynomial ring $\mathbf{Z}[x]$ of polynomials in a single variable X with coefficients in the ring of integers $\mathbf{Z}$, a principal ideal? Justify your answer.

[3a UPSC CSE 2012]

Q8. Describe the maximal ideals in the ring of Gaussian integers $\mathbf{Z}[i]=\{a+bi \mid a,b\in\mathbf{Z}\}$.

[4a UPSC CSE 2012]

Q9. How many elements does the quotient ring $\dfrac{\mathbf{Z}_5[X]}{(X^2+1)}$ have? Is it an integral domain? Justify yours answers. **[3b UPSC CSE 2009]**

Q10. How many proper, non-zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $\mathbf{Z}_{12}\oplus\mathbf{Z}_{12}$ have? Why? **[2a UPSC CSE 2009]**

CHAPTER 3. HOMOMORPHISM OF RINGS

UPDATED

Q1. Let F be a finite field of characteristic *p*, where *p* is a prime. Then show that there is an injective homomorphism from $\mathbf{Z}_p$ (group of integers modulo *p*) to F. Also show that number of elements in F is p^n, for some positive integer *n*. **[1a IFoS 2022]**

Q1. Let R be a finite field of characteristic $p(>0)$. Show that the mapping $f: R \to R$ defined by $f(a) = a^p, \forall a \in R$ is an isomorphism. **[3a UPSC CSE 2020]**

Q2. Let I and J be ideals in a ring R. Then prove that the quotient ring $(I+J)/J$ is isomorphic to the quotient ring $I/(I \cap J)$. **[2a 2019 IFoS]**

Q3. Let R be a commutative ring with unity. Prove that an ideal P of R is prime if and only if the quotient ring R/P is an integral domain. **[2d 2018 IFoS]**

Q4. If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R' prove that $\phi(1)$ is the unit element of R'. **[2a UPSC CSE 2015]**

Q5. Show that the set of matrices $S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} a, b \in \mathbf{R} \right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f: \mathbf{C} \to S$ defined by $f(a+ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here $\mathbf{R}$ is the set of real numbers and $\mathbf{C}$ is the set of complex numbers.) **[1a UPSC CSE 2013]**

Q6. Show that the quotient ring $\mathbf{Z}[i]/10\mathbf{Z}$ where $\mathbf{Z}[i]$ denotes the ring of Gaussian integers.

[3b UPSC CSE 2010]

CHAPTER 4. EUCLIDEAN RINGS, PID

UPDATED

Q1(b) Express the ideal 4*Z*+6*Z* as a principal ideal in the integral domain *Z*. **UPSC CSE 2023**

Q1. Let R be a field of real numbers and S, the field of all those polynomials $f(x) \in R[x]$ such that $f(0) = 0 = f(1)$. Prove that S is an ideal of $R[x]$. Is the residue class ring $R[x]/S$ an integral domain? Give justification for your answer. **[4a UPSC CSE 2022]**

Q1. Let *a* be an irreducible element of the Euclidean ring R, then prove that $R/(a)$ is a field.

[3d UPSC CSE 2019]

Q2. Prove that the ring $\mathbf{Z}[i]=\{a+ib:a,b\in\mathbf{Z},i=\sqrt{-1}\}$ of Gaussian integers is a Euclidean domain. **[2d 2017 IFoS]**

Q3. Show that in the ring $R=\{a+b\sqrt{-5}\,|\,a,b \text{ are integers}\}$, the elements $\alpha=3$ and $\beta=1+2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d. in R, where $\gamma=7(1+2\sqrt{-5})$.

[2c 2016 IFoS]

Q4. Let $J=\{a+bi\,|\,a,b\in\mathbf{Z}\}$ be the ring of Gaussian integers (subring of $\mathbf{C}$). What of the following is J: Euclidean domain, principal ideal domain, unique factorization domain? Justify your answer.

[3a UPSC CSE 2013]

Q5. Let R be a Euclidean domain with Euclidean valuation *d*. Let *n* be an integer such that $d(1)+n\geq 0$. Show that the function $d_n:R-\{0\}\to S$, where S is the set of all negative integers defined by $d_n(a)=d(a)+n$ for all $a\in R-\{0\}$ is a Euclidean valuation. **[4a 2010 IFoS]**

Q6. Show that $d(a)<d(ab)$, where *a*, *b* be two non-zero elements of a Euclidean domain R and *b* is not a unit in R. **[4a 2009 IFoS]**

CHAPTER 5. POLYNOMIAL RINGS, UFD UPDATED

Q1. Prove that $R[x]$ is a principal ideal domain if and only if R is a field.
[4b IFoS 2022]

Q2. Let F be a field and $f(x)\in F[x]$ a polynomial of degree > 0 over F. Show that there is a field F' and an imbedding $q:F\to F'$ s. t. the polynomial $f^q\in F'[x]$ has a root in F', where f^q is obtained by replacing each coefficient a of *f* by $q(a)$. **[2b UPSC CSE 2021]**

Q3. Show that an element x in a Euclidean domain is a unit if and only if $d(x)=d(1)$, where the notations have their usual meanings. [4b IFoS 2021]

Q3. Let R be a non-zero commutative ring with unity. If every ideal of R is prime, prove that R is a field.
(ii) Let R be a commutative ring with unity such that $a^2=a, \forall a \in R$. If I be any prime ideal of R, find all the elements of $\frac{R}{I}$. **[3b IFoS 2021]**

Q1. Let R be an integral domain with unit element. Show that any unit in $R[x]$ is a unit in R.

[1a UPSC CSE 2018]

Q2. Let F be a field and $F[X]$ denote the ring of polynomials over F in a single variable X. For $f(X), g(X) \in F[X]$ with $g(X) \neq 0$, show that there exist $q(X), r(X) \in F[X]$ such that degree $(r(X)) < \text{degree}(g(X))$ and $f(X)=q(X)\cdot g(X)+r(X)$. **[2c UPSC CSE 2017]**

Q3. Let $\mathbf{K}$ be a field and $\mathbf{K}[X]$ be the ring of polynomials over $\mathbf{K}$ in a single variable X. For a polynomial $f \in \mathbf{K}[X]$, let (f) denote the ideal in $\mathbf{K}[X]$ generated by *f*. Show that (f) is a maximal ideal in $\mathbf{K}[X]$ if and only if *f* is an irreducible polynomial over $\mathbf{K}$. **[1a UPSC CSE 2016]**

Q4. Show that every algebraically closed field is infinite. **[4a UPSC CSE 2016]**

Q5. Let R be an integral domain with unity. Prove that the units of R and $R[x]$ are same.

[3a 2014 IFoS]

Q6. If R is an integral domain, show that the polynomial ring $R[x]$ is also an integral domain.

[3c 2012 IFoS]

Q7. Let F be the set of all real valued continuous functions defined on the closed interval $[0,1]$. Prove that $(F,+,\cdot)$ is a Commutative Ring with unity with respect to addition and multiplication of functions defined point wise as below:

and $\left.\begin{aligned}(f+g)(x)&=f(x)+g(x)\\(f\cdot g)(x)&=f(x)\cdot g(x)\end{aligned}\right\} x \in [0,1]$ where $f,g \in F$. **[3a UPSC CSE 2011]**

Q8. Consider the polynomial ring $Q[x]$. Show $p(x)=x^3-2$ is irreducible over Q. Let I be the ideal in $Q[x]$ generated by $p(x)$. Then show that $Q[x]/I$ is a field and that each element of it is of the form $a_0+a_1t+a_2t^2$ with a_0,a_1,a_2 in Q and $t=x+I$. **[3a UPSC CSE 2010]**

Q9. Show that $Z[X]$ is a unique factorization domain that is not a principal ideal domain ($\mathbf{Z}$ is the ring of integers). Is it possible to give an example of principal ideal domain that is not a unique factorization domain? ($\mathbf{Z}[X]$ is the ring of polynomials in the variable X with integer.)

[3a UPSC CSE 2009]

(EXTENSION FIELD)

Q1. Let K be an extension of a field F. Prove that the elements of K, which are algebraic over F, form a subfield of K. Further, if $F \subset K \subset L$ are fields, L is algebraic over K and K is algebraic over F, then prove that L is algebraic over F. **[3a UPSC CSE 2016]**

REAL ANALYSIS TOPICS PYQs

1.REAL NUMBER SYSTEM
2.SEQUENCES
3.SERIES
    ALTERNATING SERIES
    REARRANGEMENT OF TERMS
4.UNIFORM CONVERGENCE
5.RIEMANN'S INTEGRATION
6.REAL ANALYSIS AND CALCULUS

Chapter-0 Real Number System

Q1. Let $\mathbf{R}$ denote the set of real numbers and Q denote the set of rational numbers. If $x \in \mathbf{R}, x > 0$ and $y \in \mathbf{R}$, then show that there exists a positive integer n such that $nx > y$. Use it to show that if $x < y$, then there exists $p \in Q$ such that $x < p < y$. **[1b IFoS 2022]**

Q2. Show that every open subset of $\mathbf{R}$ is countable union of disjoint open intervals.

[2d UPSC CSE 2013]

Q3. Show that a bounded infinite subset of $\mathbf{R}$ must have a limit point.

[3d UPSC CSE 2009]

CHAPTER 1. SEQUENCES

Q1. (i) Prove that every bounded and monotonically increasing sequence is convergent and converges to lub (least upper bound) of the sequence.

(ii) If $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}, \forall n \in \mathbf{N}$, then using Cauchy criterion for convergence of the sequence, show that $\{a_n\}$ is not convergent. **[3a IFoS 2022]**

Q2 Test the convergence of $\int_0^\infty \frac{\cos x}{1+x^2}\,dx$. **[1c UPSC CSE 2022]**

Q3. Apply Cauchy's Principle of Convergence to prove that the sequence $\langle f_n \rangle$ defined by

$$f_n = 1 + \frac{1}{4} + \frac{1}{7} + \ldots + \frac{1}{3n-2}$$

is not convergent. **[1b IFoS 2021]**

Q4. Examine the convergence of $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ and find its value, if possible. **[2b IFoS 2021]**

Q1. Prove that the sequence (a_n) satisfying the condition $|a_{n+1} - a_n| \le \alpha |a_n - a_{n-1}|, 0 < \alpha < 1$ for all numbers $n \ge 2$, is a Cauchy sequence.

[1c UPSC CSE 2020]

Q2. Let $x_1 = 2$ and $x_{n+1} = \sqrt{x_n + 20}$, $n = 1, 2, 3, \ldots$. Show that the sequence x_1, x_2, x_3 ..... is convergent. **[1a UPSC CSE 2017]**

Q3. Two sequences $\{x_n\}$ and $\{y_n\}$ are defined inductively by the following:

$x_1 = \frac{1}{2}, y_1 = 1$ and $x_n - \sqrt{x_{n-1} y_{n-1}}$, $n = 2, 3, 4, \ldots\ldots$

$$\frac{1}{y_n} = \frac{1}{2}\left(\frac{1}{x_n} + \frac{1}{y_{n-1}}\right), n = 2, 3, 4, \ldots.$$

Prove that

$$x_{n-1} < x_n < y_n < y_{n-1}, \quad n = 2, 3, 4, \ldots.$$

and deduce that both the sequences converge to the same limit *l*, where $\frac{1}{2} < l < 1$.

[1c UPSC CSE 2016]

Q4. Discuss the convergence of the sequence $\{x_n\}$ where $x_n = \dfrac{\sin\left(\frac{n\pi}{2}\right)}{8}$.

[1c UPSC CSE 2010]

Q5. Define $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = \sqrt{4 + x_n}$ for $n > 1$.

Show that the sequence converges to $\dfrac{(1+\sqrt{17})}{2}$. **[1d UPSC CSE 2010]**

CHAPTER 2. SERIES

Q1. Test for convergence or divergence of the series

$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \ldots (x > 0)$. **[4b UPSC CSE 2022]**

Q2. Test the uniform convergence of the series

$$x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \frac{x^4}{(1+x^4)^3} + \ldots$$

on $[0,1]$. **[1b UPSC CSE 2021]**

Q1. Find the range of $p(>0)$ for which the series:

$\frac{1}{(1+a)^p}-\frac{1}{(2+a)^p}+\frac{1}{(3+a)^p}-....,\ a>0$ is

(i) absolutely convergent and (ii) conditionally convergent. **[1d UPSC CSE 2018]**

Q2. Show that the series $\sum_{n=1}^{\infty}\left(\frac{\pi}{\pi+1}\right)^n n^6$ is convergent. **[1e UPSC CSE 2012]**

Q3. Show that the series:

$$\left(\frac{1}{3}\right)^2+\left(\frac{1\cdot 4}{3\cdot 6}\right)^2+.....+\left(\frac{1\cdot 4\cdot 7.......(3n-2)}{3\cdot 6\cdot 9.......3n}\right)^2+......$$

converges. **[2c UPSC CSE 2009]**

ALTERNATING SERIES

Q1. Show that the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$$

is conditionally convergent. (If you use any theorem(S) to show it, then you must give a proof of that theorem(s).) **[2a UPSC CSE 2016]**

Q2. Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$.

[1c UPSC CSE 2015]

REARRANGEMENT OF TERMS

Q1. Let $\sum_{n=1}^{\infty}x_n$ be a conditionally convergent series of real numbers. Show that there is a rearrangement $\sum_{n=1}^{\infty}x_{\pi(n)}$ of the series $\sum_{n=1}^{\infty}x_n$ that converges to 100. **[4c UPSC CSE 2017]**

Q2. Let $\sum_{n=1}^{\infty}a_n$ be an absolutely convergent series of real numbers.

Suppose $\sum_{n=1}^{\infty}a_{2n}=\frac{9}{8}$ and $\sum_{n=1}^{\infty}a_{2n+1}=\frac{-3}{8}$. What is $\sum_{n=1}^{\infty}a_n$?

Justify your answer. (Majority of marks is for the correct justification). **[1b 2015 IFoS]**

CHAPTER 3. CHAPTER UNIFORM CONVERGENCE

Q1. Suppose $\{f_n\}$ is a sequence of functions defined on $[a,b]$ and $\lim_{n\to\infty}f_n(x)=f(x)$, and $x\in[a,b]$. Put $M_n=\sup_{x\in[a,b]}\left|f_n(x)-f(x)\right|$.

Then show that
(i) f_n converges to f uniformly on $[a,b]$ if and only if $M_n \to 0$ as $n \to \infty$.
(ii) If $|f_n(x)| \le M_n, (x \in [a,b], n = 1, 2, ...)$, then $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[a,b]$ if $\sum_{n=1}^{\infty} M_n$ converges. **[2b IFoS 2022]**

Q1. Show that the sequence of function $\{f_n(x)\}$, where $f_n(x) = nx(1-x)^n$, does not converge uniformly on $[0,1]$. **[2b 2020 IFoS]**
Q2. Discuss the uniform convergence of
$f_n(x) = \dfrac{nx}{1+n^2x^2}, \ \forall x \in \mathbf{R}(-\infty, \infty) \ n = 1, 2, 3, ...$ **[3a UPSC CSE 2019]**
Q3. Show that the sequence $\{\tan^{-1} nx\}, x \ge 0$ is uniformly convergent on any interval $[a,b], a > 0$ but is only point wise convergent on $[0,b]$. **[3b 2019 IFoS]**
Q4. Let $f_n(x) = \dfrac{x}{n+x^2}, x \in [0,1]$. Show that the sequence $\{f_n\}$ is uniformly convergent on $[0,1]$.
[4b 2018 IFoS]
Q5. Examine the Uniform Convergence of
$f_n(x) = \dfrac{\sin(nx+n)}{n}, \forall x \in \mathbf{R}, n = 1, 2, 3, ...$ **[1b 2016 IFoS]**
Q6. If $f_n(x) = \dfrac{3}{x+n}, 0 \le x \le 2$, state with reasons whether $\{f_n\}_n$ converges uniformly on $[0,2]$ or not. **[3a 2016 IFoS]**
Q7. Test the series of functions $\sum_{n=1}^{\infty} \dfrac{nx}{(1+n^2x^2)}$ for uniform convergence. **[3b UPSC CSE 2015]**
Q8. Let $f_n(x) = \dfrac{x}{1+nx^2}$ for all real *x*. Show that f_n converges uniformly to a function *f*. What is *f*? Show that for $x \ne 0. f_n'(x) \to f'(x)$ but $f_n'(0)$ does not converge to $f'(0)$. Show that the maximum value $|f_n(x)|$ can take is $\dfrac{1}{2\sqrt{n}}$. **[3b 2015 IFoS]**
Q9. Show that the series $\sum_{1}^{\infty} \dfrac{(-1)^{n-1}}{n+x^2}$, is uniformly convergent but not absolutely for all real values of *x*. **[2c UPSC CSE 2013]**
Q10. Let

$$f_n(x)=\begin{cases} 0, & \text{if} \quad x<\dfrac{1}{n+1} \\ \sin\dfrac{\pi}{x}, & \text{if} \quad \dfrac{1}{n+1}\le x\le\dfrac{1}{n} \\ 0, & \text{if} \quad x>\dfrac{1}{n} \end{cases}$$

Show that $f_n(x)$ converges to a continuous function but not uniformly. **[1b UPSC CSE 2012]**

Q11. Examine the series

$$\sum_{n=1}^{\infty}u_n(x)=\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2}-\frac{(n-1)x}{1+(n-1)^2x^2}\right]$$

for uniform convergence. Also, with the help of this example, show that the condition of uniform convergence of $\sum_{n=1}^{\infty}u_n(x)$ is sufficient but not necessary for the sum $S(x)$ of the series to be continuous. **[4b 2012 IFoS]**

Q12. Let $f_n(x)=nx(1-x)^n$, $x\in[0,1]$

Examine the uniform convergence of $\{f_n(x)\}$ on $[0,1]$. **[2b UPSC CSE 2011]**

Q13. Show that the series for which the sum of first *n* terms

$$f_n(x)=\frac{nx}{1+n^2x^2},\ 0\le x\le 1$$

cannot be differentiated term-by-term at $x=0$. What happens at $x\neq 0$?

[3b UPSC CSE 2011]

Q14. Show that if $S(x)=\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$, then its derivative

$S'(x)=-2x\sum_{n=1}^{\infty}\frac{1}{n^2(1+nx^2)^2}$, for all *x*. **[4b UPSC CSE 2011]**

Q15. Consider the series $\sum_{n=0}^{\infty}\frac{x^2}{(1+x^2)^n}$.

Find the values of *x* for which it is convergence and also the sum function. Is the convergence uniform? Justify your answer. **[2d UPSC CSE 2010]**

Q16. Let $f_n(x)=x^n$ on $-1<x\le 1$ for $n=1,2,\ldots\ldots$ Find the limit function. Is the convergence uniform? Justify your answer. **[3c UPSC CSE 2010]**

Q17. Show that:

$$\underset{x\to 1}{Lt}\sum_{n=1}^{\infty}\frac{n^2x^2}{n^4+x^4}=\sum_{n=1}^{\infty}\frac{n^2}{n^4+1}.$$

Justify all steps of your answer by quoting the theorems you are using. **[3c UPSC CSE 2009**

CHAPTER 4. RIEMANN'S INTEGRATION

Q1. Suppose $f:[a,b]\to\mathbf{R}$ is a continuous function. Then show that f is Riemann integrable on $[a,b]$. **[1c IFoS 2022]**

Q2. Let $f(x)=x^2$ on $[0,k], k>0$. Show that f is Riemann integrable on the closed interval $[0,k]$ and $\int_0^k f\,dx=\frac{k^3}{3}$. **[2a UPSC CSE 2022]**

Q3. If a function f is monotonic in the interval $[a,b]$, then prove that f is Riemann integrable in $[a,b]$.
[1c UPSC CSE 2021]

Q1. Test the Riemann integrability of the function f defined by

$$f(x)=\begin{cases}0 & \text{when } x \text{ is rational}\\ 1 & \text{when } x \text{ is irrational}\end{cases}$$

on the interval $[0,1]$. **[1c 2019 IFoS]**

Q2. Prove the inequality: $\frac{\pi^2}{9}<\int_{\pi/6}^{\pi/2}\frac{x}{\sin x}dx<\frac{2\pi^2}{9}$. **[1b UPSC CSE 2018]**

Q3. Let

$$f(t)=\int_0^t [x]\,dx,$$

where $[x]$ denotes the largest integer less than or equal to x.

(i) Determine all the real numbers t at which f is differentiable.

(ii) Determine all the real numbers t at which f is continuous but not differentiable.

[2a UPSC CSE 2017]

Q4. Is the function $f(x)=\begin{cases}\frac{1}{n}, & \frac{1}{n+1}<x\le\frac{1}{n}\\ 0, & x=0\end{cases}$

Riemann integrable? If yes, obtain the value of $\int_0^1 f(x)dx$. **[2b UPSC CSE 2015]**

Q5. Integrate $\int_0^1 f(x)\,dx$, where

$$f(x)=\begin{cases}2x\sin\frac{1}{x}-\cos\frac{1}{x}, & x\in\,]0,1]\\ 0, & x=0\end{cases}.$$ **[2b UPSC CSE 2014]**

Q6. Let *f* be a real valued function defined on $[0,1]$ as follows:

$$f(x)=\begin{cases}\frac{1}{a^{r-1}}, \frac{1}{a^r}<x\le\frac{1}{a^{r-1}}, & r=1,2,3,....\\ 0 & x=0\end{cases}$$

where a is an integer greater than 2. Show that $\int_0^1 f(x)dx$ exists and is equal to $\frac{a}{a+1}$.

[4a 2014 IFoS P-1]

Q7. Let *f* be defined on $[0,1]$

$$f(x)=\begin{cases}\sqrt{1-x^2}, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$$

Find the upper and lower Riemann integrals of *f* over $[0,1]$. **[1b 2014 IFoS]**

Q8. Show that the function $f(x)=\sin x$ is Riemann integrable in any interval $[0,t]$ by taking the partition $P=\left\{0,\frac{t}{n},\frac{2t}{n},\frac{3t}{n},....,\frac{nt}{n}\right\}$ and $\int_0^t \sin x dx=1-\cos t$. **[4a 2014 IFoS]**

Q9. Let

$$f(x)=\begin{cases}\frac{x^2}{2}+4 & \text{if } x\ge 0\\ \frac{-x^2}{2}+2 & \text{if } x<0\end{cases}$$

Is *f* Riemann integrable in the interval $[-1,2]$? Why? Does there exist a function *g* such that $g'(x)=f(x)$? Justify your answer. **[1c UPSC CSE 2013]**

Q10. Let $[x]$ denote the integer part of the real number *x*, i.e., if $n\le x<n+1$ where n is an integer, then $[x]=n$. Is the function $f(x)=[x]^2+3$ Riemann integrable in $[-1,2]$? If not, explain why. If it is integrable, compute $\int_{-1}^{2}\left([x]^2+3\right)dx$. **[3d UPSC CSE 2013]**

Q11. Let $f(x)$ be differentiable on $[0,1]$ such that $f(1)=f(0)=0$ and $\int_0^1 f^2(x)dx=1$. Prove that $\int_0^1 xf(x)f'(x)dx=-\frac{1}{2}$. **[3b UPSC CSE 2012]**

Q12. Give an example of a function $f(x)$, that is not Riemann integrable but $|f(x)|$ is Riemann integrable. Justify. **[4b UPSC CSE 2012]**

Q13. Determine whether

$$f(x)=2x\sin\frac{1}{x}\cos\frac{1}{x}$$

is Riemann-integrable on $[0,1]$ and justify your answer. **[1c 2011 IFoS]**

Q14. Let the function *f* be defined by

$f(x)=\frac{1}{2^t}$, when $\frac{1}{2^{t+1}}<x\le\frac{1}{2^t}$ $(t=0,1,2,3,....)$

$f(0)=0$

If *f* integrable on $[0,1]$? If *f* is integrable, then evaluate $\int_0^1 f\,dx$. **[2b 2011 IFoS]**

Q15. Show that the function $f(x)=[x^2]+|x-1|$ is Riemann integrable in the interval $[0,2]$ where $[\alpha]$ denote the greatest integer less than or equal to α. Can you give an example of a function that is not Riemann integrable on $[0,2]$? Compute $\int_0^2 f(x)dx$, where $f(x)$ is as above. **[1f P-1 UPSC CSE 2010]**

REAL ANALYSIS AND CALCULUS

LIMIT, CONTINUITY & DIFFERENTIABILITY

Q1. Show that if a function *f* defined on an open interval (a,b) of $\mathbf{R}$ is convex, then *f* is continuous. Show, by example, if the condition of open interval is dropped, then the convex function need not be continuous. **[2c UPSC CSE 2018]**

Q2. Suppose $\mathbf{R}$ be the set of all real numbers and $f:\mathbf{R}\to\mathbf{R}$ is a function such that the following equations hold for all $x,y\in\mathbf{R}$:

(i) $f(x+y)=f(x)+f(y)$

(ii) $f(xy)=f(x)f(y)$

Show that $\forall x\in\mathbf{R}$ either $f(x)=0$ or $f(x)=x$. **[4a UPSC CSE 2018]**

Q3. A function $f:[0,1]\to[0,1]$ is continuous on $[0,1]$. Prove that there exists a point *c* in $[0,1]$ such that $f(c)=c$. **[1b 2018 IFoS]**

Q4. Find the supremum and the infimum of $\frac{x}{\sin x}$ on the interval $\left(0,\frac{\pi}{2}\right]$.

[1c UPSC CSE 2017]

Q5. A function $f:\mathbf{R}\to\mathbf{R}$ is defined as below:

$$f(x)=\begin{cases} x \text{ if } x \text{ is rational} \\ 1-x \text{ if } x \text{ is irrational}\end{cases}$$

Prove that *f* is continuous at $x=\frac{1}{2}$ but discontinuous at all other point in $\mathbf{R}$.

[1b 2017 IFoS]

Q6. For the function $f:(0,\infty)\to\mathbf{R}$ given by

$f(x)=x^2\sin\frac{1}{x}, 0<x<\infty$

show that there is a differentiable function $g:\mathbf{R}\to\mathbf{R}$ that extends *f*. **[1b UPSC CSE 2016]**

Q7. Examine the continuity of $f(x,y)=\begin{cases}\dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)}, & (x,y)\neq(0,0)\\ \dfrac{1}{2}, & (x,y)=(0,0)\end{cases}$ at the point $(0,0)$.

[3b 2016 IFoS]

Q8. Answer the following:
(a) Show that the function $f:\mathbf{R}\to\mathbf{R}$ defined by

$f(x)=\begin{cases}1, & x \text{ is irrational}\\ -1, & x \text{ is rational}\end{cases}$

is discontinuous at every point in $\mathbf{R}$. **[1a 2012 IFoS]**

Q9. Define the function

$f(x)=x^2\sin\frac{1}{x}$, if $x\neq 0$

$=0$, if $x=0$

Find $f'(x)$. Is $f'(x)$ continuous at $x=0$? Justify your answer. **[2c UPSC CSE 2010]**

Q10. If $f:\mathbf{R}\to\mathbf{R}$ is such that

$f(x+y)=f(x)f(y)$

for x,y in $\mathbf{R}$ and $f(x)\neq 0$ for any *x* in $\mathbf{R}$, show that $f'(x)=f(x)$ for all *x* in $\mathbf{R}$ given that $f'(0)=f(0)$ and the function is differentiable for all *x* in $\mathbf{R}$. **[1c 2010 IFoS]**

Q11. Show that if $f:[a,b]\to\mathbf{R}$ is a continuous function then $f([a,b])=[c,d]$ for some real numbers *c* and *d*, $c\le d$. **[2d UPSC CSE 2009]**

UNIFORM CONTINUITY

Q1. Prove that the function $f(x)=\sin x^2$ is *not* uniformly continuous on the interval $[0,\infty[$.

[2b UPSC CSE 2020]

Q2. Show that the function $f(x)=\sin\left(\frac{1}{x}\right)$ is continuous and bounded in $(0,2\pi)$, but it is not uniformly continuous in $(0,2\pi)$. **[1b 2019 IFoS]**

Q3. Let $f:\mathbf{R}\to\mathbf{R}$ be a continuous function such that $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ exist and are finite. Prove that *f* is uniformly continuous on $\mathbf{R}$. **[4b UPSC CSE 2016]**

Q4. Let $X=(a,b]$. Construct a continuous function $f:X\to\mathbf{R}$ (set of real numbers) which is unbounded and not uniformly continuous on *X*. Would your function be uniformly continuous on $[a+\varepsilon,b], a+\varepsilon<b$? Why? **[2b 2015 IFoS]**

Q5. Show that the function $f(x)=\sin\frac{1}{x}$ is continuous but not uniformly continuous on $(0,\pi)$.

[2b IFoS 2014]

Q6. Show that the function $f(x)=x^2$ is uniformly continuous in $\langle 0,1\rangle$ but not in $\mathbf{R}$.

[3a (13m) 2013 IFoS]

Q7. Let $S=(0,1]$and f be defined by $f(x)=\frac{1}{x}$ where $0<x\le 1$ (in $\mathbf{R}$). Is *f* uniformly continuous on *S*? Justify your answer. **[1b UPSC CSE 2011]**

DIFFERENTIATION UNDER INTEGRAL SIGN

Q1. Evaluate

$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx,\ a>0,\ a\neq 1$. **[1c UPSC CSE 2019]**

PDE

1. FORMATION OF PDE
2. FIRST ORDER LINEAR PDE- LAGRANGE'S METHOD
   ORTHOGONAL TRAJECTORIES
3. FIRST ORDER NON- LINEAR PDE- CHARPIT'S METHOD, SOME STANDARD FORMS
   CAUCHY'S CHARACTERISTIC METHOD
4. LINEAR PDE WITH CONSTANT COEFFICIENTS-HOMOGENEOUS, NON-HOMOGENEOUS
5. CLASSIFICATION OF PDEs AND REDUCTION IN CANONICAL FORMS
6. METHOD OF SEPARATION OF VARIABLES
   WAVE EQUATION
   HEAT EQUATION
   LAPLACE EQUATION
   MISCELLANEOUS

CHAPTER 1. FORMATION OF PDE

BY ELIMINATION OF ARBITRARY CONSTANTS

Q1. Find the partial differential equation of the family of all tangent planes to the ellipsoid: $x^2+4y^2+4z^2=4$, which are not perpendicular to the *xy*-plane. **[5a UPSC CSE 2018]**

Q2. Find the partial differential equation of all planes which are at a constant distance *a* from the origin. **[(5d) 2018 IFoS]**

Q3. Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and *v* be the velocity at a distance *x* from a fixed point at time *t*, then $\frac{\partial^2 \rho}{\partial t^2}=\frac{\partial^2}{\partial x^2}\left\{\rho\left(v^2+k\right)\right\}$. **[(5d) 2018 IFoS]**

Q4. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2+y^2+z^2=cz$. **[5a UPSC CSE 2016]**

BY ELIMINATION OF ARBITRARY FUNCTIONS

Q1. Obtain the partial differential equation by eliminating arbitrary function *f* from the equation $f\left(x+y+z, x^2+y^2+z^2\right)=0$. **[5a UPSC CSE 2021]**

Q1. Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z=y(f)+xg(y)$ and specify its nature (elliptic, hyperbolic or parabolic in the region $x>0, y>0$. **[5a UPSC CSE 2020]**

Q2. Construct a partial differential equitation of all surfaces of revolution having the *z*-axis as the axis of rotation. **[(5a) 2020 IFoS]**

Q3. Form a partial differential equation of the family of surfaces given by the following expression: $\psi\left(x^2+y^2+2z^2, y^2-2zx\right)=0$. **[1a UPSC CSE 2019]**

Q4. Form the partial differential equitation by eliminating arbitrary functions φ and ψ from the relation $z=\varphi\left(x^2-y\right)+\psi\left(x^2+y\right)$. **[(5a) 2017 IFoS]**

Q5. Obtain the partial differential equation governing the equations

$\phi(u,v)=0,\ u=xyz$

$v=x+y+z$. **[(5a) 2016 IFoS]**

Q6. Form a partial differential equation by eliminating the arbitrary functions *f* and *g* from $z=yf(x)+xg(y)$. **[5a UPSC CSE 2013]**

Q7. Eliminate the arbitrary function *f* from the given equation

$f\left(x^2+y^2+z^2, x+y+z\right)=0$. **[(5b) 2013 IFoS]**

CHAPTER 2. FIRST ORDER LINEAR PDE- LAGRANGE'S METHOD

Q1. Find the integral surface of the partial differential equitation:
$(x-y)y^2\frac{\partial z}{\partial x}+(y-x)x^2\frac{\partial z}{\partial y}=\left(x^2+y^2\right)z$ that contains the curve: $xz=a^3,\ y=0$ on it.

[6a UPSC CSE 2020]

Q2. Find the general solution of the partial differential equation $p\tan x+q\tan y=\tan z$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. **[(5e) 2020 IFoS]**

Q3. Find the general solution of the partial differential equation:
$\left(y^3x-2x^4\right)p+\left(2y^4-x^3y\right)q=9z\left(x^3-y^3\right)$, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x=t, y=t^2, z=1$. **[6a UPSC CSE 2018]**

Q4. Solve $\left(z^2-2yz-y^2\right)p+(xy+zx)q=xy-zx$, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$. If the solution of the above equation represents a sphere, what will be the coordinates of its centre?

[(7a) UPSC CSE 2018]

Q5. Solve the partial differential equation:
$(x-y)\frac{\partial z}{\partial x}+(x+y)\frac{\partial z}{\partial y}=2xz$. **[(6a) UPSC CSE 2017]**

Q6. Find the general integral of the partial differential equation $(y+zx)p-(x+yz)q=x^2-y^2$.

[5e UPSC CSE 2016]

Q7. Find the general solution of the partial differential equation
$xy^2\frac{\partial z}{\partial x}+y^3\frac{\partial z}{\partial y}=\left(zxy^2-4x^3\right)$. **[(5b) UPSC CSE 2016]**

Q8. Find the general solution of the partial differential equation $xy^2p+y^3q=\left(zxy^2-4x^3\right)$
$\left[p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}\right]$. **[(6a) UPSC CSE 2016]**

Q9. Solve the partial differential equation $\left(y^2+z^2-x^2\right)p-2xyq+2xz=0$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. **[5a UPSC CSE 2015]**

Q10. Solve for the general solution $p\cos(x+y)+q\sin(x+y)=z$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

[6a UPSC CSE 2015]

Q11. Solve the PDE:
$xu_x+yu_y+zu_z=xyz$. **[(6a) UPSC CSE 2013]**

Q12. Solve the partial differential equation $px+qy=3z$. **[6a UPSC CSE 2012]**

Q13. Solve $\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy$ using Lagrange's Method. **[(8a) UPSC CSE 2012]**

Q14. Solve the PDE
$(x+2z)\frac{\partial z}{\partial x}+(4zx-y)\frac{\partial z}{\partial y}=2x^2+y$. **[5b UPSC CSE 2011]**

Q15. Find the general solution of $x\left(y^2+z\right)p+y\left(x^2+z\right)q=z\left(x^2-y^2\right)$. **[(5a) 2010 IFoS]**

ORTHOGONAL TRAJECTORIES

Q1. Find the system of equations for obtaining the general equation of surfaces orthogonal to the family given by
$x\left(x^2+y^2+z^2\right)=Cy^2$, where C is a parameter. **[6a IFoS 2022]**

Q2. Find the orthogonal trajectory of the following family of curves:
$x^2-y^2=a^2$
Then sketch the two families to demonstrate whether they cut orthogonally. **[5e IFoS 2021]**

Q1. Find the equations of the system of curves on the cylinder $2y=x^2$ orthogonal to its intersection with the hyperboloids of the one-parameter system $xy=z+c$. **[(7c) 2019 IFoS]**

Q2. Find the surface which is orthogonal to the family of surfaces $z(x+y)=c(3z+1)$ and which passes through the circle $x^2+y^2=1, z=1$. **[(6b) 2017 IFoS]**

Q3. Find the surface which intersects the surfaces of the system $z(x+y)=C(3z+1)$, (C being a constant) orthogonally and which passes through the circle $x^2+y^2=1, z=1$. **[6b UPSC CSE 2013]**

MISCELLANEOUS

Q1. Test the integrability of the equation $z\left(z+y^2\right)dx+z\left(z+x^2\right)dy-xy\left(x+y\right)dz=0$. If integrable, then find its solution. **[(6a) 2019 IFoS]**

Q2. Verify that the differential equation $\left(y^2+yz\right)dx+\left(xz+z^2\right)dy+\left(y^2-xy\right)dz=0$ is integrable and find its primitive. **[(6a) 2014 IFoS]**

Q3. Find the surface satisfying $\frac{\partial^2 z}{\partial x^2}=6x+2$ and touching $z=x^3+y^3$ along its section by the plane $x+y+1=0$. **[6a UPSC CSE 2011]**

CHAPTER 4. FIRST ORDER NON- LINEAR PDE- CHARPIT'S METHOD, SOME STANDARD FORMS

Q1. Find a complete integral of the partial differential equation

$$\left(p^2+q^2\right)x=pz;\ p=\frac{\partial z}{\partial x},q=\frac{\partial z}{\partial y}$$

using Charpit's method and hence deduce the solution which passes through the curve $x=0, z^2=4y$.

[8c IFoS 2022]

Q2. Solve the following by Charpit's method:

$pxy+pq+qy=yz,\ p=\frac{\partial z}{\partial x},q=\frac{\partial z}{\partial y}$. **[6a IFoS 2021]**

Q3 Solve the following differential equation:

$\left(y^2+z^2-x^2\right)p-2xyq+2xz=0,\ p=\frac{\partial z}{\partial x},q=\frac{\partial z}{\partial y}$. **[7c IFoS 2021]**

Q4. Find a complete integral of the partial differential equation $p=(z+qy)^2$ by using Charpit's method. **[8a UPSC CSE 2021]**

Q1. Find the general solution and singular solution of the partial differential equation $6yz-6pxy-3qy^2+pq=0$. **[(6a) 2020 IFoS]**

Q2. Find a complete integral of the equation by Charpit's method $p^2x+q^2y=z$. Here $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$. **[(5e) 2019 IFoS]**

Q3. Find the complete integral of the partial differential equation $\left(p^2+q^2\right)x=zp$ and deduce the solution which passes through the curve $x=0, z^2=4y$. Here $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$. **[(6a) 2018 IFoS]**

Q4. Find a complete integral of the partial differential equation $2\left(pq+yp+qx\right)+x^2+y^2=0$.

[6a UPSC CSE 2017]

Q5. Find complete integral of $xp-yq=xqf\left(z-pz-qy\right)$ where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$.

[(6c) UPSC CSE 2017]

Q6. Find the solution of the equation

$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=1$ the passes through the circle $x^2+y^2=1, u=1$. **[(7c) UPSC CSE 2013]**

CAUCHY'S CHARACTERISTIC METHOD

Q1. Find the solution of the partial differential equation

$z=\frac{1}{2}\left(p^2+q^2\right)+\left(p-x\right)\left(q-y\right); p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$

which passes through the *x*-axis, using Cauchy's method of characteristics. **[7c IFoS 2022]**

Q1. Find the solution of the partial differential equation:

$z=\frac{1}{2}\left(p^2+q^2\right)+\left(p-x\right)\left(q-y\right);\ p\equiv\frac{\partial z}{\partial x},\ q\equiv\frac{\partial z}{\partial y}$ which passes through the *x*-axis.

[7a UPSC CSE 2020]

Q2. Solve the first order quasilinear partial differential equation by the method of characteristics:

$x\frac{\partial u}{\partial x}+\left(u-x-y\right)\frac{\partial u}{\partial y}=x+2y$ in $x>0, -\infty<y<\infty$ with $u=1+y$ on $x=1$.

[6a UPSC CSE 2019]

Q3. Determine the characteristics of the equation $z=p^2-q^2$, and find the integral surface which passes through the parabola $4z+x^2=0,\ y=0$. **[6a UPSC CSE 2016]**

Q4. Solve the following partial differential equation $zp+yq=x$

$x_0\left(s\right)=x, y_0\left(s\right)=1, z_0\left(s\right)=2s$ by the method of characteristics. **[6a UPSC CSE 2010]**

CHAPTER 5. LINEAR PDE WITH CONSTANT COEFFICIENTS HOMOGENEOUS

Q1. Find the general solution of the partial differential equation
$\left(D^2+DD'-6D'^2\right)z=x^2\sin\left(x+y\right)$
where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$. **[7a UPSC CSE 2022]**

Q1. Solve the partial differential equation:
$\left(D^3-2D^2D'-DD'^2+2D'^3\right)z=e^{2x+y}+\sin\left(x-2y\right)$;
$D\equiv\frac{\partial}{\partial x},D'\equiv\frac{\partial}{\partial y}$. **[5d UPSC CSE 2020]**

Q2. Find the solution of the following differential equation:
$2\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x\partial y}+3\frac{\partial^2 z}{\partial y^2}=ye^x$. **[(7c) 2020 IFoS]**

Q3. Find the solution of the equation:
$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$. **[(5a) 2019 IFoS]**

Q4. Solve the partial differential equation:
$\left(2D^2-5DD'+2D'^2\right)z=5\sin\left(2x+y\right)+24\left(y-x\right)+e^{3x+4y}$ where $D\equiv\frac{\partial}{\partial x},D'\equiv\frac{\partial}{\partial y}$.
[7a UPSC CSE 2018]

Q5. Find a real function V of x and y, satisfying $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=-4\pi\left(x^2+y^2\right)$ and reducing to zero, when $y=0$. **[(8a) 2018 IFoS]**

Q6. Solve $\left(D^2-2DD'+D'^2\right)z=e^{x+2y}+x^3+\sin 2x$ where $D\equiv\frac{\partial}{\partial x},D'\equiv\frac{\partial}{\partial y},D^2\equiv\frac{\partial^2}{\partial x^2},D'^2\equiv\frac{\partial^2}{\partial y^2}$. **[5a UPSC CSE 2017]**

Q7. Solve the partial differential equation
$\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}-\frac{\partial^3 z}{\partial x\partial y^2}+2\frac{\partial^3 z}{\partial y^3}=e^{x+y}$. **[7a UPSC CSE 2016]**

Q8. Find the particular integral of $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=2x\cos y$. **[(6b) 2016 IFoS]**

Q9. Solve $\left(D^2+DD'-2D'^2\right)u=e^{x+y}$, where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$. **[5b UPSC CSE 2015]**

Q10. Find the solution of the equation $u_{xx}-3u_{xy}+u_{yy}=\sin\left(x-2y\right)$. **[(5d) 2015 IFoS]**

Q11. Solve the partial differential equation $\left(2D^2-5DD'+2D'^2\right)z=24\left(y-x\right)$.

[5a UPSC CSE 2014]

Q12. Solve $\left(D^2+DD'-6D'^2\right)z=x^2\sin\left(x+y\right)$ where D and D' denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$.

[6a UPSC CSE 2013]

Q13. Solve the partial differential equitation $\left(D-2D'\right)\left(D-D'\right)^2 z=e^{x+y}$. **[5a UPSC CSE 2012]**

Q14. Solve $\left(D^3D'^2+D^2D'^3\right)z=0$ where D stands for $\frac{\partial}{\partial x}$ and D' stands for $\frac{\partial}{\partial y}$.
[(5b) 2012 IFoS]

Q15. Find the complementary function and particular integral of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$.

[(7b) 2011 IFoS]

Q16. Solve the PDE $\left(D^2-D'\right)\left(D-2D'\right)Z=e^{2x+y}+xy$. **[5a UPSC CSE 2010]**

Q17. Find the surface satisfying the PDE $\left(D^2-2DD'+D'^2\right)Z=0$ and the conditions that $bZ=y^2$ when $x=0$ and $aZ=x^2$ when $y=0$. **[5b UPSC CSE 2010]**

NON-HOMOGENEOUS

Q1. Find the general solution of the partial differential equation
$\left(D^2-D'^2-3D+3D'\right)z=xy+e^{x+2y}$
where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$. **[7a UPSC CSE 2021]**

Q1. Solve: $\left(D-3D'-2\right)^2 z=2e^{2x}\cot\left(y+3x\right)$. **[(7a) 2014 IFoS]**

Q2. Solve the PDE $\left(D^2-D'^2+D+3D'-2\right)z=e^{(x+y)}-x^2y$. **[5a UPSC CSE 2011]**

Q3. Find the general solution of $\left(D-D'-1\right)\left(D-D'-2\right)z=e^{2x-y}+\sin\left(3x+2y\right)$.

[(7a) 2010 IFoS]

CHAPTER 5. CLASSIFICATION AND REDUCTION IN CANONICAL FORMS

Q1. Reduce the following partial differential equation to a canonical form and hence solve it:

$yu_{xx}+(x+y)u_{xy}+xu_{yy}=0$. **[8a UPSC CSE 2022]**

Q1. Reduce the following second order partial differential equation to canonical form and find the general solution:

$$\frac{\partial^2 u}{\partial x^2}2x\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial y}+12x$$. **[7c UPSC CSE 2019]**

Q2. Reduce the equation

$$y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}.$$

to canonical form and hence solve it. **[7a UPSC CSE 2017]**

Q3. Reduce the second-order partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$

into canonical form. Hence, find its general solution. **[8a UPSC CSE 2015]**

Q4. Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form. **[6a UPSC CSE 2014]**

Q5. Reduce the equation

$$y\frac{\partial^2 z}{\partial x^2}+(x+y)\frac{\partial^2 z}{\partial x\partial y}+x\frac{\partial^2 z}{\partial y^2}=0$$

to its canonical form when $x\neq y$. **[5b UPSC CSE 2013]**

Q6. Rewrite the hyperbolic equation $x^2u_{xx}-y^2u_{yy}=0\,(x>0, y>0)$ in canonical form.

[(6c) 2013 IFoS]

Q7. Reduce the equation

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$$

to its canonical form and solve. **[(5a) 2011 IFoS]**

Q8. Reduce the following 2nd order partial differential equation into canonical form and find its general solution $xu_{xx}+2x^2u_{xy}-u_x=0$. **[6b UPSC CSE 2010]**

CHAPTER 6. METHOD OF SEPARATION OF VARIABLES

Q1. Find the solution of the initial-boundary value problem

$u_t-u_{xx}+u=0,\ 0<x<l,\ t>0$

$u(0,t)=u(l,t)=0,\ t\geq 0$

$u(x,0)=x(l-x),\ 0<x<l$. **[7a UPSC CSE 2015]**

Q2. Solve the differential equation $u_x^2-u_y^2$ by variable separation method. **[(6b) 2015 IFoS]**

Q3. Show that the general solution of the pde

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

is of the form $Z(x,y) = F(x+ct) + G(x-ct)$, where F and G are arbitrary functions.

[(5c) 2014 IFoS]

WAVE EQUATION

Q1. Solve the wave equation

$$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L,\ t > 0$$

subject to the conditions

$$u(0,t) = 0,\ u(L,t) = 0$$

$u(x,0) = \frac{1}{4}x(L-x), \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$. **[6a UPSC CSE 2021]**

Q1. One end of a tightly stretched flexible thin string of length l is fixed at the origin and the other at $x = l$. It is plucked at $x = \frac{l}{3}$ so that it assumes initially the shape of a triangle of height h in the $x-y$ plane. Find the displacement y at any distance x and at any time t after the string is released from rest. Take, $\frac{\text{horizontal tension}}{\text{mass per unit length}} = c^2$. **[8a UPSC CSE 2020]**

Q2. Given the one-dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2};\ t > 0,$$

where $c^2 = \frac{T}{m}$, T is the constant tension in the string and m is the mass per unit length of the string.

(i) Find the appropriate solution of the above wave equation.

(ii) Find also the solution under the conditions $y(0,t) = 0,\ y(l,t) = 0$ for all t and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0, y(x,0) = a\sin\frac{\pi x}{l},\ 0 \le x \le l, a > 0$. **[8a UPSC CSE 2017]**

Q3. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0\sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position, find the displacement $y(x,t)$.

[(6d) 2017 IFoS]

Q4. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection

$$u(x,0)=\begin{cases}\frac{2}{l}x, & \text{if } 0<x<\frac{l}{2}\\ \frac{2}{l}(l-x), & \text{if } \frac{l}{2}\le x<l\end{cases}$$

with initial velocity $u_t(x,0)=0$. **[(8c) 2015 IFoS]**

Q5. Find the deflection of a vibrating string (length $=\pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$. **[7a UPSC CSE 2014]**

Q6. Solve $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0<x<1, t>0$, given that

(i) $u(x,0)=0,\ 0\le x\le 1$

(ii) $\frac{\partial u}{\partial t}(x,0)=x^2,\ 0\le x\le 1$

(iii) $u(0,t)=u(1,t)=0$, for all *t*. **[8a UPSC CSE 2014]**

Q7. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in equilibrium position. If it is set vibrating by giving end point a velocity $\lambda.x(l-x)$, find the displacement of the string at any distance *x* from one end at any time *t*. **[6c UPSC CSE 2013]**

Q8. A string of length *l* is fixed at its ends. The string from the mid-point is pulled up to a height *k* and then released from rest. Find the deflection $y(x,t)$ of the vibrating string. **[6b UPSC CSE 2012]**

Q9. A uniform string of length *l* is held fixed between the points $x=0$ and $x=l$. The two points of trisection are pulled aside through a distance ε on opposite sides of the equilibrium position and is released from rest at time $t=0$. Find the displacement of the string at any latter time $t>0$. What is the displacement of the string at the midpoint? **[(6a) UPSC CSE 2011]**

HEAT EQUATION

Q1. Solve the heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<l, t>0$ subject to the conditions

$u(0,t)=u(l,t)=0$

$u(x,0)=x(l-x), 0\le x\le l$. **[6a UPSC CSE 2022]**

Q1. A thin annulus occupies the region $0<a\le r\le b, 0\le\theta\le 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0°, while along

the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$, where K is a constant. Determine the temperature distribution in the annulus.

[8c UPSC CSE 2018]

Q2. Find the temperature $u(x,t)$ in a bar of silver of length 10 cm and constant cross-section of area 1 cm^2. Let density $\rho = 10.6$ g/cm^3, thermal conductivity K = 1.04 cal / (cm sec°C) and specific heat $\sigma = 0.56$ cal/g°C. The bar is perfecty isolated laterally, with ends kept at 0°C and initial temperature $f(x) = \sin(0.1\pi x)$ °C. Note that $u(x,t)$ follows the heat equation $u_t = c^2 u_{xx}$, where $c^2 = K/(\rho\,\sigma)$. **[8a UPSC CSE 2016]**

Q3. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

[UPSC CSE (6c) 2016]

Q4. Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1, t > 0$$

subject to the conditions $u(0,t) = u(1,t) = 0$ for $t > 0$ and $u(x,0) = \sin \pi x,\ 0 < x < 1$.

[UPSC CSE (7a) 2015]

Q5. Solve the following heat equation, using the method of separation of variables:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1, t > 0$$

subject to the conditions

$u = 0$ at $x = 0$ and $x = 1$, for $t > 0$

$u = 4x(1-x)$, at $t = 0$ for $0 \le x \le 1$. **[(8a) 2013 IFoS]**

Q6. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state.

[7b UPSC CSE 2012]

Q7. Obtain temperature distribution $y(x,t)$ in a uniform bar of unit length whose one end is kept at 10°C and the other end is insulated. Also it is given that $y(x,0) = 1 - x,\ 0 < x < 1$.

[6c UPSC CSE 2011]

Q8. Solve the following heat equation

$u_t - u_{xx} = 0,\ 0 < x < 2,\ t > 0$

$u(0,t) = u(2,t) = 0,\ t > 0$

$u(x,0) = x(2-x),\ 0 \le x \le 2$.

[6c UPSC CSE 2010]

Q9. Solve

$$\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$$

given the conditions

(i) $u(0,t)=u(\pi,t)=0,\ t>0$

(ii) $u(x,0)=\sin 2x,\ 0<x<\pi$. **[(6c) UPSC CSE 2010]**

LAPLACE EQUATION

Q1. Let Γ be a closed curve in *xy*-plane and let S denote the region bounded by the curve Γ. Let

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=f(x,y)\forall(x,y)\in S.$$

If f is prescribed at each point (x,y) of S and w is prescribed on the boundary Γ of S, then prove that any solution $w=w(x,y)$, satisfying these conditions, is unique. **[5d UPSC CSE 2017]**

Q2. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ subject to the conditions $u(0,y)=u(l,y)=u(x,0)=0$ and $u(x,a)=\sin\left(\frac{n\pi x}{l}\right)$. **[(8d) 2017 IFoS]**

Q3. Using Method of Separation of variables, Solve Laplace Equation in three dimensions.

[(6a) 2012 IFoS]

Q4. Solve

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ 0\le x\le a,\ 0\le y\le b$$

satisfying the boundary conditions

$u(0,y)=0,\ u(x,0)=0,\ u(x,b)=0$

$\frac{\partial u}{\partial x}(a,y)=T\sin^3\frac{\pi y}{a}$. **[6b UPSC CSE 2011]**

COMPLEX ANALYSIS

1.ANALYTIC FUNCTIONS
2.CONFORMAL & BILINEAR TRANSFORMATIONS
3.COMPLEX INTEGRATION
CAUCHY INTEGRAL FORMULA
4.TAYLOR'S AND LAURENT'S SERIES
5. SINGULARITIES, RESIDUES, CAUCHY'S RESIDUE THEOREM
6.CONTOUR INTEGRATION

CHAPTER 1. ANALYTIC FUNCTIONS

Q1. If $f(z)=u+iv$ is any analytic function of the complex variable z and $u-v=e^{x}(\cos y-\sin y)$, find $f(z)$ in terms of z. **[1e IFoS 2021]**

Q1. If $v(r,\theta)=\left(r-\frac{1}{r}\right)\sin\theta,\ r\neq 0$ then find an analytic function $f(z)=u(r,\theta)+iv(r,\theta)$.

[4a UPSC CSE 2020]

Q2. Suppose $f(z)$ is analytic function on a domain D in ϕ and satisfies the equation $\operatorname{Im} f(z)=(\operatorname{Re} f(z))^{2}, Z\in D$. Show that $f(z)$ is constant in D. **[1d UPSC CSE 2019]**

Q3. If $f(z)$ is analytic in a domain D and $|f(z)|$ is a non-zero constant in D, then show that $f(z)$ is constant in D. **[2c 2019 IFoS]**

Q4. Prove that the function $u(x,y)=(x-1)^{3}-3xy^{2}+3y^{2}$ is harmonic and find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z. **[1c UPSC CSE 2018]**

Q5. If $u=(x-1)^{3}-3xy^{2}+3y^{2}$, determine v so that $u+iv$ is a regular function of $x+iy$.

[1c 2018 IFoS]

Q6. Let $f=u+iv$ be an analytic function on the unit disc $D=\{z\in\mathbf{C}:|z|<1\}$. Show that

$$\frac{\partial^{2}u}{\partial x^{2}}+\frac{\partial^{2}u}{\partial y^{2}}=0=\frac{\partial^{2}v}{\partial x^{2}}+\frac{\partial^{2}v}{\partial y^{2}}$$

at all points of D. **[3b UPSC CSE 2017]**

Q7. If $f(z)=u(x,y)+iv(x,y)$ is an analytic function of $z=x+iy$ and $u+2v=x^{3}-2y^{3}+3xy(2x-y)$ then find $f(z)$ in terms of z. **[1c 2017 IFoS]**

Q8. Is $v(x,y)=x^{3}-3xy^{2}+2y$ a harmonic function? Prove your claim. If yes, find its conjugate harmonic function $u(x,y)$ and hence obtain the analytic function whose real and imaginary parts are u and v respectively. **[1d UPSC CSE 2016]**

Q9. Find the analytic function of which the real part is $e^{-x}\{(x^{2}-y^{2})\cos y+2xy\sin y\}$. **[1d 2016 IFoS]**

Q10. Show that the function $v(x,y)=\ln(x^2+y^2)+x+y$ is harmonic. Find its conjugate harmonic function $u(x,y)$. Also, find the corresponding analytic function $f(z)=u+iv$ in terms of z.

[1d UPSC CSE 2015]

Q11. Let $u(x,y)=\cos x\sinh y$. Find the harmonic conjugate $v(x,y)$ of u and express $u(x,y)+iv(x,y)$ as a function of $z=x+iy$. **[1c 2015 IFoS]**

Q12. Prove that the function $f(z)=u+iv$, where

$$f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2},\ z\neq 0;\ f(0)=0$$

satisfies Cauchy-Riemann equations at the origin, but the derivative of f at $z=0$ does not exist.

[1c UPSC CSE 2014]

Q13. Find the constants a,b,c such that the function

$$f(z)=2x^2-2xy-y^2+i\{ax^2-bxy+cy^2\}$$

is analytic for all $z=x+iy$ and express $f(z)$ in terms of z. **[2e 2014 IFoS]**

Q14. Construct an analytic function

$f(z)=u(x,y)+iv(x,y)$, where $v(x,y)=6xy-5x+3$

Express the result as a function of z. **[1c 2013 IFoS]**

Q15. Show that the function defined by

$$f(z)=\begin{cases}\dfrac{x^3y^5(x+iy)}{x^6+y^{10}}, & z\neq 0\\ 0, & z=0\end{cases}$$

is not analytic at the origin though it satisfies Cauchy-Riemann equations at the origin.

[1c UPSC CSE 2012]

Q16. Show that the function

$u(x,y)=e^{-x}(x\cos y+y\sin y)$

is harmonic. Find its conjugate harmonic function $v(x,y)$ and the corresponding analytic function $f(z)$. **[3b 2012 IFoS]**

Q17. If $f(z)=u+iv$ is an analytic function of $z=x+iy$ and $u-v=\dfrac{e^y-\cos x+\sin x}{\cosh y-\cos x}$, find $f(z)$ subject to the condition, $f\left(\dfrac{\pi}{2}\right)=\dfrac{3-i}{2}$. **[1c UPSC CSE 2011]**

Q18. Show that $u(x,y)=2x-x^3+3xy^2$ is a harmonic function. Find a harmonic conjugate of $u(x,y)$. Hence find the analytic function f for which $u(x,y)$ is the real part.

[1e UPSC CSE 2010]

Q19. Determine the analytic function $f(z)=u+iv$ if $v=e^x(x\sin y+y\cos y)$.

[1d 2010 IFoS]

CHAPYTER 2. CONFORMAL MAPPINGS & BILINEAR TRANSFORMATIONS

Q1. Find a bilinear transformation $w = f(z)$ which maps the upper half plane $\mathrm{Im}(z) \geq 0$ onto the unit disk $|w| \leq 1$. **[2c IFoS 2022]**

Q1. Show that the bilinear transformation

$$w = e^{i\theta_0}\left(\frac{z - z_0}{z - \bar{z}_0}\right)$$

z_0 being in the upper half of the *z*-plane, maps the upper half of the *z*-plane into the interior of the unit circle in the w-plane. If under this transformation, the point $z = i$ is mapped into $w = 0$ while the point at infinity is mapped into $w = -1$, then find this transformation. **[4a 2020 IFoS]**

Q2. Find the bilinear transformations which map the point $-1, \infty, i$ into the points -

(i) $i, 1, 1+i$

(ii) $\infty, i, 1$

(iii) $0, \infty, 1$ **[3c 2014 IFoS]**

Q3. Sketch the image of the infinite strip $1 < y < 2$ under the transformation $w = \dfrac{1}{z}$.

[2c 2011 IFoS]

Q4. Find the image of the finite vertical strip $R: x = 5$ to $x = 9, -\pi \leq y \leq \pi$ of *z*-plane under exponential function. **[4a UPSC CSE 2010]**

CHAPTER 3. COMPLEX INTEGRATION

Q1. Compute the integral

$$\oint_C \frac{1 + 2z + z^2}{(z-1)^2 (z+2)} dz$$

where C is $|z| = 3$. **[1e IFoS 2022]**

Q2.(i) Find an upper bound for the absolute value of the integral $I=\int_C e^z dz$, where C is the line segment joining the points $(0,0)$ and $(1,3)$. **[4a IFoS 2022]**

Q1. Evaluate the integral $\int_C (z^2+3z)dz$ counterclockwise from $(2,0)$ to $(0,2)$ along the curve C, where C is the circle $|z|=2$. **[1d UPSC CSE 2020]**

Q2. Evaluate the integral $\int_C \operatorname{Re}(z^2)dz$ from 0 to $2+4i$ along the curve C where C is a parabola $y=x^2$. **[3c UPSC CSE 2019]**

Q3. Evaluate the integral $\int_C \operatorname{Re}(z^2)dz$ from 0 to $2+4i$ along the curve $C: y=x^2$.

[1e 2019 IFoS]

Q4. Evaluate the integral

$$\int_{2-i}^{4+i}\left(x+y^2-ixy\right)dz$$

along the line segment AB joining the points $A(2,-1)$ and $B(4,1)$. **[1c 2012 IFoS]**

Q5. Evaluate the line integral $\int_C f(z)dz$ where $f(z)=z^2, c$ is the boundary of the triangle with vertices $A(0,0), B(1,0), C(1,2)$ in that order. **[4a UPSC CSE 2010]**

CAUCHY INTEGRAL FORMULA

Q1. Using Cauchy theorem and Cauchy integral formula, evaluate the integral $\oint_C \frac{e^z}{z^2(z+1)^3}dz$ where C is $|z|=2$. **[2c 2020 IFoS]**

Q2. Using Cauchy's integral formula, evaluate the integral $\oint_C \frac{dz}{(z^2+4)^2}$ where $c:|z-i|=2$.

[1d 2019 IFoS]

Q3. Using Cauchy integral formula, evaluate

$$\int_C \frac{z+2}{(z+1)^2(z-2)}dz$$

where C is the circle $|z-i|=2$. **[1c 2014 IFoS]**

Q4. Use Cauchy integral formula to evaluate $\int_C \frac{e^{3z}}{(z+1)^4}dz$, where c is the circle $|z|=2$.

[2c UPSC CSE 2012]

CHAPTER 4. TAYLOR'S AND LAURENT'S SERIES , SINGULARITIES

Q1. Obtain the first three terms of the Laurent series expansion of the function $f(z)=\frac{1}{(e^z-1)}$ about the point $z=0$ valid in the region $0<|z|<2\pi$. **[4b UPSC CSE 2019]**

Q2. Classify the singular point $z=0$ of the function $f(z)=\frac{e^z}{z+\sin z}$ and obtain the principal part of the Laurent series expansion of $f(z)$. **[4b 2019 IFoS]**

Q3. Find the Laurent's series which represent the function $\frac{1}{(1+z^2)(z+2)}$ when

(i) $|z|<1$

(ii) $1<|z|<2$

(iii) $|z|>2$ **[4b UPSC CSE 2018]**

Q4. Determine all entire function $f(z)$ such that 0 is a removable singularity of $f\left(\frac{1}{z}\right)$.

[1d UPSC CSE 2017]

Q5. For a function $f:\mathbf{C}\to\mathbf{C}$ and $n\geq 1$, let $f^{(n)}$ denote the n^{th} derivative of f and $f^{(0)}=f$. Let f be an entire function such that for some $n\geq 1$, $f^{(n)}\left(\frac{1}{k}\right)=0$ for all $k=1,2,3,....$ Show that f is a polynomial. **[4a UPSC CSE 2017]**

Q6. Prove that every power series represents an analytic function inside its circle of convergence.

[4c UPSC CSE 2016]

Q7. Find the Laurent series for the function $f(z)=\frac{1}{1-z^2}$ with centre $z=1$.

[4b 2016 IFoS]

Q8. Find all possible Taylor's and Laurent's series expansions of the function $f(z)=\frac{2z-3}{z^2-3z+2}$ about the point $z=0$. **[2c UPSC CSE 2015]**

Q9. Expand in Laurent series the function $f(z)=\frac{1}{z^2(z-1)}$ about $z=0$ and $z=1$.

[1d UPSC CSE 2014]

Q10. Find the Laurent series expansion at $z=0$ for the function

$$f(z)=\frac{1}{z^2(z^2+2z-3)}$$

in the region (i) $1<|z|<3$ and (ii) $|z|>3$. **[4b 2014 IFoS]**

Q11. Prove that if $be^{a+1}<1$ where a and b are positive and real, then the function $z^n e^{-a}-be^z$ has n zeroes in the unit circle. **[1d UPSC CSE 2013]**

Q12. Find Laurent series about the indicated singularity. Name the singularity and give the region of convergence.

$\frac{z-\sin z}{z^3}; z=0$. **[4b 2013 IFoS]**

Q13. Expand the function $f(z)=\frac{1}{(z+1)(z+3)}$ in Laurent series valid for

(i) $1<|z|<3$ (ii) $|z|<3$ (iii) $0<|z+1|<2$ (iv) $|z|<1$ **[3c UPSC CSE 2012]**

Q14. Find the Laurent series for the function

$f(z)=\frac{1}{1-z^2}$ with centre $z=1$. **[3d UPSC CSE 2011]**

Q15. If the function $f(z)$ is analytic and one valued in $|z-a|<R$, prove that for $0<r<R$, $f'(a)=\frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}d\theta$, where $P(\theta)$ is the real part of $f(a+re^{i\theta})$.

[2c UPSC CSE 2011]

Q16. Expand the function

$$f(z)=\frac{2z^2+11z}{(z+1)(z+4)}$$

in a Laurent's series valid for $2<|z|<3$. **[1d 2011 IFoS]**

Q17. Find the Laurent series of the function

$f(z)=\exp\left[\frac{\lambda}{2}\left(z-\frac{1}{z}\right)\right]$ as $\sum_{n=-\infty}^{\infty} C_n z^n$ for $0<|z|<\infty$ where $C_n=\frac{1}{\pi}\int_0^{\pi}\cos(n\phi-\lambda\sin\phi)\,d\phi$, $n=0,\pm1,\pm2,....$ with λ a given complex number and taking the unit circle C given by $z=e^{i\phi}\,(-\pi\le\phi\le\pi)$ as contour in this region. **[4b UPSC CSE 2010]**

Q18. Obtain Laurent's series expansion of the function

$$f(z)=\frac{1}{(z+1)(z+3)}$$

in the region $0<|z+1|<2$. **[4b 2010 IFoS]**

CHAPTER 5. RESIDUES, CAUCHY'S RESIDUE THEOREM

Q1. Show that an isolated singular point z_0 of a function $f(z)$ is a pole of order m if and only if $f(z)$ can be written in the form $f(z)=\frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and non zero at z_0. Moreover $\operatorname*{Res}_{z=z_0} f(z)=\frac{\phi^{(m-1)}(z_0)}{(m-1)!}$ if $m\geq 1$. **[2d UPSC CSE 2019]**

Q2. Show by applying the residue theorem that $\int_0^\infty \frac{dx}{(x^2+a^2)^2}=\frac{\pi}{4a^3}$, $a>0$.

[3b UPSC CSE 2018]

Q3. Prove that $\int_0^\infty \cos x^2 dx=\int_0^\infty \sin x^2 dx=\frac{1}{2}\sqrt{\frac{\pi}{2}}$. **[2c 2018 IFoS]**

Q4. Evaluate the integral $\int_0^{2\pi}\cos^{2n}\theta d\theta$, where n is a positive integer. **[3c 2018 IFoS]**

Q5. Find the sum of residues of $f(z)=\frac{\sin z}{\cos z}$ at its poles inside the circle $|z|=2$.

[4b 2017 IFoS]

Q6. State Cauchy's residue theorem. Using it, evaluate the integral

$\int_C \frac{e^z+1}{z(z+1)(z-i)^2}dz; C:|z|=2$. **[3a UPSC CSE 2015]**

Q7. Evaluate the integral $\int_r \frac{z^2}{(z^2+1)(z-1)^2}dz$, where r is the circle $|z|=2$.

[2c 2015 IFoS]

Q8. Evaluate the integral $\int_0^\pi \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$ using residues. **[3c UPSC CSE 2014]**

Q9. Evaluate:

$\int_{|z|=1}\frac{z}{z^4-6z^2+1}dz$. **[2c 2014 IFoS]**

Q10. Using Cauchy's residue theorem, evaluate the integral

$I=\int_0^\pi \sin^4\theta d\theta$. **[4b UPSC CSE 2013]**

Q11. Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$ where c is the circle $|z|=3$. **[3c 2013 IFoS]**

Q12. Using the Residue Theorem, evaluate the integral

$\int_C \frac{e^z-1}{z(z-1)(z+i)^2}dz$. **[4a 2012 IFoS]** where C is the circle $|z|=2$.

Q13. State Cauchy's residue theorem. Using it, evaluate the integral

$\int_C \frac{e^{z/2}}{(z+2)(z^2-4)}dz$. **[4b 2011 IFoS]**

counterclockwise around the circle $C:|z+1|=4$.

CHAPTER 6. CONTOUR INTEGRATION

Q1. Using contour integration, evaluate the integral $\int_0^{2\pi}\frac{1}{3+2\sin\theta}d\theta$. **[2c UPSC CSE 2020]**

Q2. Using contour integral method, prove that

$\int_0^{\infty}\frac{x\sin mx}{a^2+x^2}dx=\frac{\pi}{2}e^{-ma}$. **[2b UPSC CSE 2017]**

Q3. Prove by the method of contour integration that $\int_0^{\pi}\frac{1+2\cos\theta}{5+4\cos\theta}d\theta=0$.

[4a 2017 IFoS]

Q4. Let

$\gamma:[0,1]\to\mathbf{C}$ be the curve

$\gamma(t)=e^{2\pi it},\ 0\le t\le 1$.

Find, giving justifications, the value of the contour integral

$\int_{\gamma}\frac{dz}{4z^2-1}$. **[3c UPSC CSE 2016]**

Q5. Evaluate by Contour integration $\int_0^{\pi}\frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$. **[4c 2016 IFoS]**

Q6. Show that $\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dx=\frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem.

[4b 2015 IFoS]

Q7. Evaluate by contour integration

$I=\int_0^{2\pi}\frac{d\theta}{1-2a\cos\theta+a^2}, a^2<1$. **[3d UPSC CSE 2012]**

Q8. Evaluate by Contour integration,

$\int_0^1\frac{dx}{(x^2-x^3)^{1/3}}$. **[3c UPSC CSE 2011]**

Q9. Using the method of contour integration, evaluate

$\int_{-\infty}^{\infty} \frac{x^2 dx}{\left(x^2+1\right)^2\left(x^2+2x+2\right)}$. **[2c 2010 IFoS]**

NUMERICAL ANALYSIS

1.INTERPOLATION

WITH EQUAL INTERVALS

WITH UNEQUAL INTERVALS

2.NUMERICAL INTEGRATION

3.NUMERICAL DIFFERENTIATION

4.SOLVING ALGEBRAIC & TRANSCENDENTAL EQUATIONS

5.SYSTEM OF LINEAR EQUATIONS

6.BOOLEAN ALGEBRA & COMPUTER PROGRAMING

7.LOGARITHMS AND FLOW CHART

CHAPTER 1. INTERPOLATION
NEWTON'S (FORWARD AND BACKWARD) INTERPOLATION

Q1. From the following table, estimate the number of students who obtained marks between 40 and 46:

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	32	43	55	40	30

[1b IFoS 2021]

Q2. Derive Newton's backward difference interpolation formula and also do error analysis.

[8b UPSC CSE 2021]

Q1. The following table gives the values of $y = f(x)$ for certain equidistant values of *x*. Find the value of $f(x)$ when $x = 0.612$ using Newton's forward difference interpolation formula.

x:	0.61	0.62	0.63	0.64	0.65
$y = f(x)$	1.840431	1.858928	1.877610	1.896481	1.915541

[5b 2019 IFoS]

Q2. Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1, u_2 = 9, u_3 = 25, u_4 = 55$ and $u_5 = 105$. **[5b UPSC CSE 2018]**

Q3. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. **[5c UPSC CSE 2013]**

WITH UNEQUAL INTERVALS

Q1. Given $f(1) = 4, f(2) = 5, f(7) = 5$ and $f(8) = 4$. Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum. **[1b IFoS 2022]**

Q1. Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data:

x:	0.1	0.2	0.3	0.4
y:	0.9975	0.9900	0.9776	0.9604

[7a UPSC CSE 2012]

Q2. The velocity of a particle at time t is as follows:

t(seconds)	0	2	4	6	8	10	12
v (m/sec)	4	6	16	36	60	94	136

Find its displacement at the 12th second and acceleration at the 2nd second.

[7c 2011 IFoS]

LAGRANGE'S INTERPOLATION

Write the three point Lagrangian interpolating polynomial relative to the points $x_0, x_0+\varepsilon$ and x_1. Then by taking the limit $\varepsilon \to 0$, establish the relation

$$f(x)=\frac{(x_1-x)(x+x_1-2x_0)}{(x_1-x_0)^2}f(x_0)+\frac{(x-x_0)(x_1-x)}{(x_1-x_0)}f'(x_0)+\frac{(x-x_0)^2}{(x_1-x_0)}f(x_1)+E(x)$$

where $E(x)=\frac{1}{6}(x-x_0)^2(x-x_1)f'''(\xi)$ is the error function and min. $(x_0, x_0+\varepsilon, x_1)<\xi$ max. $(x_0, x_0+\varepsilon, x_1)$. **[8b UPSC CSE 2020]**

Q2. Find the Lagrange interpolating polynomial that fits the following data values:

x:	-1	2	3	5
$f(x)$:	-1	10	25	60

Also, interpolate at $x=2.5$, correct to three decimal places. **[6b 2020 IFoS]**

Q3. For given equidistant values u_{-1}, u_0, u_1 and u_2 a value is interpolated by Lagrange's formula. Show that it may be written in the form

$$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$$

where $x+y=1$. **[6b UPSC CSE 2017]**

Q4. Let $f(x)=e^{2x}\cos 3x$, for $x\in[0,1]$. Estimate the value of $f(0.5)$ using Lagrange interpolating polynomial of degree 3 over the nodes $x=0$, $x=0.3, x=0.6$ and $x=1$. Also, compute the error bound over the interval $[0,1]$ and the actual error $E(0.5)$. **[6c UPSC CSE 2016]**

Q5. Apply Lagrange's interpolation formula to find $f(5)$ and $f(6)$ given that $f(1)=2, f(2)=4, f(3)=8, f(7)=128$. **[5d 2016 IFoS]**

Q6. Find the Lagrange interpolating polynomial that fits the following data:

x:	-1	2	3	4
$f(x)$:	-1	11	31	69

Find $f[1\cdot 5]$. **[6c UPSC CSE 2015]**

Q7. Show that $\sum_{k=1}^{n} l_k(x)=1$, where $l_k(x), k=1$ to n, are Lagrange's fundamental polynomials.

[5b 2015 IFoS]

Q8. Use Lagrange's formula to find the form of $f(x)$ from the following table:

x:	0	2	3	6
$f(x)$	648	704	729	792

[5a 2014 IFoS]

Q9. The values of $f(x)$ for different values of x are given as $f(1)=4, f(2)=5, f(7)=5$ and $f(8)=4$. Using Lagrange's interpolation formula, find the value of $f(6)$. Also find the value of x for which $f(x)$ is optimum. **[6c 2014 IFoS]**

Q10. Using Lagrange's interpolation formula, show that
$32f(1)=-3f(-4)+10f(-2)+30f(2)-5f(4)$. **[5a 2012 IFoS]**

Q11. For the data

x:	0	1	2	5
$f(x)$:	2	3	12	147

find the cubic function of x. **[5b 2011 IFoS]**

Q12. For the given set of data points $(x_1, f(x_1)),(x_2, f(x_2)),....(x_n, f(x_n))$ write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula. **[7c (ii) UPSC CSE 2010]**

Q13. Using Lagrange interpolation, obtain an approximate value of $\sin(0.15)$ and a bound on the truncation error for the given data $\sin(0.1)=0.09983, \sin(0.2=0.19867)$. **[6a 2010 IFoS]**

Q14. Find the interpolating polynomial for $(0,2),(1,3),(2,12)$ and $(5,147)$. **[7c 2010 IFoS]**

CHAPTER 3. NUMERICAL INTEGRATION
TRAPEZOIDAL RULE

Q1. Evaluate the integral $\int_0^2 \frac{x}{1+x^3}dx$, using trapezoidal rule with $h=\frac{1}{4}$, correct to three decimal places. (h is the length of subinterval). **[8b 2020 IFoS]**

COMPOSITE TRAPEZOIDAL RULE

Q1. Evaluate $\int_0^1 e^{-x^2}dx$ using the composite trapezoidal rule with four decimal precision, i.e., with the absolute value of the error not exceeding 5×10^{-5}. **[5d 2017 IFoS]**

SIMPSON'S 1/3rd RULE

Q1.The velocity of a train which starts from rest is given by the following table, the time being reckoned in minutes from the start and the velocity in km/hour:

t (minutes)	2	4	6	8	10	12	14	16	18	20
v (km/hour)	16	28.8	40	46.4	51.2	32	17.6	8	3.2	0

Using Simpson's $\frac{1}{3}$rd rule, estimate approximately in km the total distance run in 20 minutes.

[6B upsc cse 2022]

Q1. Following values of x_i and the corresponding values of y_i are given. Find $\int_0^3 y\,dx$ using Simpson's one-third rule.

x_i :	0.0	0.5	1.0	1.5	2.0	2.5	3.0
y_i	0.0	0.75	1.0	0.75	0.0	-1.25	-3.0

[5c 2019 IFoS]

Q2. Starting from rest in the beginning, the speed (in Km/h) of a train at different times (in minutes) is given by the below table:

Time (Minutes)	2	4	6	8	10	12	14	16	18	20
Speed (Km/h)	10	18	25	29	32	20	11	5	2	8.5

Using Simpson's $\frac{1}{3}$rd rule, find the approximate distance travelled (in Km) in 20 minutes from the beginning. **[5d UPSC CSE 2018]**

Q3. The velocity v (km/min) of a moped is given at fixed interval of time (min) as below:

t	0.1	0.2	0.3	0.4	0.5	0.6
v	1.00	1.104987	1.219779	1.34385	1.476122	1.615146

t	0.7	0.8	0.9	1.0	1.1
v	1.758819	1.904497	2.049009	2.18874	2.31977

Estimate the distance covered during the time (use Simpson's one-third rule). **[7b 2018 IFoS]**

Q4. A river is 80 metre wide, the depth y, in metre, of the river at a distance x from one bank is given by the following table:

X	0	10	20	30	40	50	60	70	80

Y	0	4	7	9	12	15	14	8	3

Find the area of cross-section of the river using Simpson's $\frac{1}{3}$rd rule. **[7b 2017 IFoS]**

Q5. Evaluate $\int_0^{0.6} \frac{dx}{\sqrt{1-x^2}}$ by Simpson's $\frac{1}{3}$ rd rule, by taking 12 equal sub-intervals.

[7a 2016 IFoS]

Q6. The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule.

[7c UPSC CSE 2013]

Q7. A river is 80 meters wide. The depth *d* (in meters) of the river at a distance x from one bank of the river is given by the following table:

X	0	10	20	30	40	50	60	70	80
D	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section of the river. **[6c 2012 IFoS]**

Q8. Calculate $\int_2^{10} \frac{dx}{1+x}$ (upto 3 places of decimal) by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$rd Rule. **[5c UPSC CSE 2011]**

Q9. A solid of revolution is formed by rotating about the *x*-axis, the area between the *x*-axis, the line $x=0$ and $x=1$ and a curve through the points with the following co-ordinates:

x	.00	.25	.50	.75	1
y	1	.9896	.9589	.9089	.8415

Find the volume of the solid. **[7a UPSC CSE 2011]**

Q10. The velocity of a particle at time *t* is as follows:

t (seconds):	0	2	4	6	8	10	12
v (m/sec)	4	6	16	36	60	94	136

Find its displacement at the 12th second and acceleration at the 2nd second. **[7c 2011 IFoS]**

Q11. Find the value of the integral

$$\int_1^5 \log_{10} x dx$$

by using Simpson's $\frac{1}{3}$-rule correct up to 4 decimal places. Take 8 subintervals in your computation.

[7b UPSC CSE 2010]

SIMPSON'S 3/8 RULE

Q1. Derive the formula

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\ldots+y_{n-1})+2(y_3+y_6+\ldots+y_{n-3})\right]$$

Is there any restriction on n? State that condition. What is the error bound in the case of Simpson's $\frac{3}{8}$ rule? **[7b UPSC CSE 2017]**

GAUSSIAN QUADRATURE FORMULA

Q1. Find a quadrature formula

$$\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$

which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (correct up to three decimal places). **[7b UPSC CSE 2020]**

Q2. Use Gauss quadrature formula of point six to evaluate $\int_0^1 \frac{dx}{1+x^2}$ given

$x_1 = -0.23861919, \quad w_1 = 0.46791393$

$x_2 = -0.66120939 \quad w_2 = 0.36076157$

$x_3 = -0.93246951, \quad w_3 = 0.17132449$

$x_4 = -x_1, x_5 = -x_2, x_6 = -x_3, w_4 = w_1, w_5 = w_2$ and $w_6 = w_3$. **[8c 2019 IFoS]**

Q3. Find the values of the constant a,b,c such that the quadrature formula $\int_0^h f(x)\,dx = h\left[af(0)+bf\left(\frac{h}{3}\right)+cf(h)\right]$ is exact for polynomials of as high degree as possible and hence find the order of the truncation error. **[7b UPSC CSE 2018]**

Q4. For an integral $\int_{-1}^{1} f(x)\,dx$, show that the two-point Gauss quadrature rule is given by $\int_{-1}^{1} f(x)\,dx = f\left(\frac{1}{\sqrt{3}}\right)+f\left(-\frac{1}{\sqrt{3}}\right)$. Using this rule, estimate $\int_2^4 2xe^x\,dx$. **[7c UPSC CSE 2016]**

WEDDLE'S RULE

Q1. A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x=0$ and a curve through the points with the following coordinates:

x	0.0	0.25	0.50	0.75	1.00	1.25	1.50
y	1.0	0.9896	0.9589	0.9089	0.8415	0.8415	0.7635

Estimate the volume of the solid formed using Weddle's rule. **[5b 2018 IFoS]**

CHAPTER 4. NUMERICAL DIFFERENTIATION
EULER'S METHOD

Q1**12**. Use Euler's method with step size $h=0.15$ to compute the approximate value of $y(0.6)$, correct up to five decimal places from the initial value problem
$y'=x(y+x)-2$
$y(0)=2$. **[7b UPSC CSE 2013]**

MODIFIED EULER'S METHOD

Q1. Find y for $x=0.2$ taking $h=0.1$ by modified Euler's method and compute the error, given that: $\frac{dy}{dx}=x+y,\ y(0)=1$. **[7c 2017 IFoS]**

Q2. Using Euler's Modified Method, obtain the solution of
$\frac{dy}{dx}=x+\left|\sqrt{y}\right|, \quad y(0)=1$
for the range $0\le x\le 0.6$ and step size 0.2. **[8c 2012 IFoS]**

RUNGE KUTTA METHOD

Q1. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$. Use four decimal places for calculation and step length 0.2. **[5d UPSC CSE 2019]**

Q2. Given $\frac{dy}{dx}=x^2+y^2, y(0)=1$. Find $y(0.1)$ and $y(0.2)$ by fourth order Runge-Kutta method.

[7a 2019 IFoS]

Q3. Apply fourth-order Runge-Kutta method to compute y at $x=0.1$ and $x=0.2$ given that $\frac{dy}{dx}=x+y^2, y=1$ at $x=0$. **[6b 2018 IFoS]**

Q4. Use the Classical Fourth-order Runge-Kutta method with $h=.2$ to calculate a solution at $x=.4$ for the initial value problem $\frac{dy}{dx}=x+y^2$ with initial condition $y=1$ when $x=0$.

[7c 2016 IFoS]

Q5. Solve the initial value problem $\frac{dy}{dx}=x(y-x), y(2)=3$ in the interval $[2,2.4]$ using the Runge-Kutta fourth-order method with step size $h=0.2$. **[7b UPSC CSE 2015]**

Q6. Use the classical fourth order Runge-Kutta methods to find solutions at $x=0.1$ and $x=0.2$ of the differential equation $\frac{dy}{dx}=x+y, y(0)=1$ with step size $h=0.1$. **[7c 2015 IFoS]**

Q7. Use Runge-Kutta formula of fourth order to find the value of y at $x=0.8$, where $\frac{dy}{dx}=\sqrt{x+y}$, $y(0.4)=0.41$. Take the step length $h=0.2$. **[6c UPSC CSE 2014]**

Q8. Using Runge-Kutta 4th order method, find y from

$$\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$$

with $y(0)=1$ at $x=0.2,0.4$. **[8a 2014 IFoS]**

Q9. Use the Classical Fourth-order Runge-Kutta method with $h=0.2$ to calculate a solution at $x=0.4$ for the initial value problem $\frac{du}{dx}=4-x^2+u, u(0)=0$ on the interval $[0,0.4]$.

[8b 2013 IFoS]

Q10. Solve the initial value problem

$$\frac{dy}{dx}=\frac{y-x}{y+x}, \; y(0)=1$$

for $x=0.1$ by Euler's method. **[8b 2010 IFoS]**

CHAPTER 5. SOLUTIONS OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS

REGULA-FALSI

Q1. Using Regula-Falsi method, find the fourth root of 28 correct to three decimal places.
[6b IFoS2021]

Q1. The equation $x^6-x^4-x^3-1=0$ has one real root between 1.4 and 1.5. Find the root to four places of decimal by regula-falsi method. **[8b 2018 IFoS]**

Q2. Solve $x\log_{10}x=1.2$ by regula falsi method. **[5b 2010 IFoS]**

NEWTON-RAPHSON METHOD

Q1. Find the real root of the equation $e^x-3x=0$, by Newton-Raphson method, correct up to four decimal places. **[8b IFoS 2022]**

Q2. Find a positive root of the equation $3x=1+\cos x$ by a numerical technique using initial values $0,\frac{\pi}{2}$; and further improve the result using Newton-Raphson method correct to 8 significant figures.

[5b UPSC CSE 2021]

Q1. Show that the equation: $f(x)=\cos\frac{\pi(x+1)}{8}+0.148x-0.9062=0$ has one root in the interval $(-1,0)$ and one in $(0,1)$. Calculate the negative root correct to four decimal places using Newton-Raphson method. **[5b UPSC CSE 2020]**

Q2. Using Newton-Raphson method, find the value of $(37)^{1/3}$, correct to four decimal places.

[5b 2020 IFoS]

Q3. Apply Newton-Raphson method, to find a real root of transcendental equation $x\log_{10}x=1.2$, correct to three decimal places. **[5b UPSC CSE 2019]**

Q4. Find the real root of the equation $x^3+x^2+3x+4=0$ correct up to five places of decimal using Newton- Raphson method. **[7a 2017 IFoS]**

Q5. Find the cube root of 10 up to 5 significant figures by Newton-Raphson method.

[7b 2016 IFoS]

Q6. Apply Newton-Raphson method to determine a root of the equation $\cos x-xe^x=0$ correct up to four decimal places. **[5b UPSC CSE 2014]**

Q7. Use Newton-Raphson method and derive the iteration scheme $x_{n+1}=\frac{1}{2}\left(x_n+\frac{N}{x_n}\right)$ to calculate an approximate value of the square root of a number N. Show that the formula $\sqrt{N}\approx\frac{A+B}{4}+\frac{N}{A+B}$ where $AB=N$, can easily be obtained if the above scheme is applied two times. Assume $A=1$ as an initial guess value and use the formula twice to calculate the value of $\sqrt{2}$ [For 2nd iteration, one may take A = rest of the 1st iteration] **[5a 2013 IFoS]**

Q8. Use Newton-Raphson method to find the real root of the equation $3x=\cos x+1$ correct to four decimal places. **[5b UPSC CSE 2012]**

Q9. Find the smallest positive root of the equation $x^3-6x+4=0$ correct to four decimal places using Newton-Raphson method. From this root, determine the positive square root of 3 correct to four decimal places. **[6c 2011 IFoS]**

Q10. Find the positive root of the equation $10xe^{-x^2}-1=0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[5c UPSC CSE 2010]**

CHAPTER 5. SYSTEM OF LINEAR EQUATIONS

GAUSSIAN ELIMINATION (DIRECT)

Q1. Solve, by Gauss elimination method, the system of equations

$2x+2y+4z=18$

$x+3y+2z=13$

$3x+y+3z=14$. **[5b UPSC CSE 2022]**

Q1. Solve the following system of linear equations using Gaussian elimination method:

$5x_1 + 2x_2 + x_3 = -2$

$6x_1 + 3x_2 + 2x_3 = 1$

$x_1 - x_2 + 2x_3 = 0$. **[8c 2020 IFoS]**

Q2. Solve the following system of equations:

$2x_1 + x_2 + x_3 - 2x_4 = -10$

$4x_1 + 2x_3 + x_4 = 8$

$3x_1 + 2x_2 + 2x_3 = 7$

$x_1 + 3x_2 + 2x_3 - x_4 = -5$. **[7b 2014 IFoS]**

GAUSS JORDON METHOD (DIRECT)

Q1. Solve the following system of equations by Gauss-Jordan method:

$2x + y - 3z = 11$

$4x - 2y + 3z = 8$

$-2x + 2y - z = -6$. **[7a IFoS 2022]**

Q1. Solve the following system of equations by Gauss-Jordan elimination method:

$x_1 + x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 6$

$x_1 - x_2 - x_3 = -3$. **[6b 2019 IFoS]**

Q2. Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix

$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$. **[5b UPSC CSE 2017]**

GAUSS JACOBI METHOD (ITERATIVE)

Q1. Solve by Gauss-Jacobi method of iteration the equations

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

[correct to two decimal places] **[5c 2011 IFoS]**

GAUSS- SEIDEL METHOD (ITERATIVE)

Q1. Solve the following system of equations by Gauss-Seidel method:

$20x + y - 3z = 16$

$2x + 20y - z = -19$

$3x - 2y + 20z = 25$

starting with the initial solution $x_0 = y_0 = z_0 = 0$. **[8b IFoS 2021]**

Q1. For the solution of the system of equations:

$4x + y + 2z = 4$

$3x + 5y + z = 7$

$x + y + 3z = 3$

set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{(0)} = 0$. Also find the exact solutions and compare with the iterated solutions.

[6b UPSC CSE 2020]

Q2. Apply Gauss-Seidel iteration method to solve the following system of equations:

$2x + y - 2z = 17$,

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$, correct to three decimal places. **[7b UPSC CSE 2019]**

Q3. Find the solution of the system

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

using Gauss-Seidel method (make four iterations). **[8b UPSC CSE 2015]**

Q4. Solve the following system of linear equations correct to two places by Gauss-Seidel method:

$x + 4y + z = -1,\ 3x - y + z = 6,\ x + y + 2z = 4$. **[6a 2015 IFoS]**

Q5. Solve the system of equations

$2x_1 - x_2 = 7$

$-x_1 + 2x_2 - x_3 = 1$

$-x_2 + 2x_3 = 1$

using Gauss-Seidel iteration method (Perform three interations). **[6b UPSC CSE 2014]**

Q6. Solve the following system of simultaneous equations, using Gauss-Seidel iterative method:

$3x + 20y - z = -18$

$20x + y - 2z = 17$

$2x - 3y + 20z = 25$. **[6c UPSC CSE 2012]**

Q7. Solve the following system of equations using Gauss-Seidel Method:

$28x + 4y - z = 32$

$2x + 17y + 4z = 35$

$x + 3y + 10z = 24$

correct to three decimal places. **[7b 2012 IFoS]**

CHAPTER 7. NUMBER SYSTEMS & BOLLEAN ALGEBRA

NUMBER SYSTEMS & OPERATIONS

Q1.(i) Convert the number $(1093 \cdot 21875)_{10}$ into octal and the number $(1693 \cdot 0628)_{10}$ into hexadecimal systems.
(ii) Express the Boolean function $F(x, y, z) = xy + x'z$ in a product of maxterms form.
[5c UPSC CSE 2022]
Q2. (i) Convert $(3798 \cdot 3875)_{10}$ into octal and hexadecimal equivalents.
(ii) Obtain the principal conjunctive normal form of $(\quad P \rightarrow R) \wedge (Q \,\square\, P)$. **[5c USC CSE 2021]**

Q1. Answer the following questions:
(i) Convert $(14231)_8$ into an equivalent binary number and then find the equivalent decimal number.
(ii) Convert $(43503)_{10}$ into an equivalent binary number and then find the equivalent hexadecimal number. **[5c 2020 IFoS]**
Q2. Find the equivalent numbers given in a specified number to the system mentioned against them:
(i) Integer 524 in binary system
(ii) 101010110101.101101011 to octal system
(iii) decimal number 5280 to hexadecimal system
(iv) Find the unknown number $(1101.101)_8 \rightarrow (?)10$. **[6a UPSC CSE 2019]**
Q3. Find the equivalent of numbers given in a specified number system to the system mentioned against them.
(i) $(111011.101)_2$ to decimal system
(ii) $(1000111110000.00101100)_2$ to hexadecimal system
(iii) $(C4F2)_{16}$ to decimal system
(iv) $(418)_{10}$ to binary system. **[6b UPSC CSE 2018]**
Q4. Assuming a 16-bit computer representation of signed integers, represent -44 in 2's complement representation. **[7c 2018 IFoS]**
Q5. Assuming a 32 bit computer representation of signed integers using 2's complement representation, add the two numbers -1 and -1024 and give the answer in 2's complement representation. **[7d 2017 IFoS]**
Q6. Convert the following decimal numbers to equivalent binary and hexadecimal numbers:
(i) 4096
(ii) 0.4375
(iii) 2048.0625 **[5d UPSC CSE 2016]**

Q7. Store the value of -1 in hexadecimal in a 32-bit computer. **[5a 2015 IFoS]**
Q8. Convert $(0.231)_5, (104.231)_5$ and $(247)_7$ to base 10. **[6b 2013 IFoS]**
Q9. Compute $(3205)_{10}$ to the base 8. **[5d(i) UPSC CSE 2011]**
Q10. Find the hexadecimal equivalent of the decimal number $(587632)_{10}$. **[7c(i) UPSC CSE 2010]**
Q11. Convert the following:
(i) $(736.4)_8$ to decimal number
(ii) $(41.6875)_{10}$ to binary number
(iii) $(101101)_2$ to decimal number
(iv) $(AF63)_{16}$ to decimal number
(v) $(101111011111)_2$ to hexadecimal number. **[5c 2010 IFoS]**

BOOLEAN ALGEBRA- LOGIC GATES, TRUTH TABLES, NORMAL FORMS, LOGIC CIRCUITS

Q1. If $x = 0.101010101E0001010$ and $y = 0.100010110E0000110$, then find $x - y$.
(ii) Draw the map of the Boolean function $F = x'yz + xy'z' + xyz + xyz'$. Also simplify the function.
[5c IFoS 2022]

Q2. Find a combinatorial circuit corresponding to the Boolean function
$f(x, y, z) = \left[x \cdot (\bar{y} + z) + y\right]$
and write the input/output table for the circuit. **[6b UPSC CSE 2022]**

Q3. Consider the following integers and their 8-bits binary representations:
13=00001101, 20=00010100
Perform the following bitwise operations and express the results in decimal system:
(i) 13 & 20 (Bitwise AND)
(ii) 13|20 (Bitwise OR)
(iii) 13^20 (Bitwise XOR)
(iv) ~20 (Bitwise Compliment). **[5c IFoS 2021]**

Q4. Obtain the Boolean function $F(x,y,z)$ based on the table given below. Then simplify $F(x,y,z)$ and draw the corresponding GATE network: **[6b UPSC CSE 2021]**

x	y	z	$F(x,y,z)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Q1. Let $g(w,x,y,z)=(w+x+y)(x+\bar{y}+z)(w+\bar{y})$ be a Boolean function. Obtain the conjunctive normal for $g(w,x,y,z)$. Also express $g(w,x,y,z)$ as a product of maxterms.

[5c UPSC CSE 2020]

Q2. Given the Boolean expression

$X = AB + ABC + \overline{AB}\,\overline{C} + A\overline{C}$

(i) Draw the logical diagram for the expression

(ii) Minimize the expression

(iii) Draw the logical diagram for the reduced expression. **[8a UPSC CSE 2019]**

Q3. Simplify the boolean expression:

$(a+b)\cdot(\bar{b}+c)+b\cdot(\bar{a}+\bar{c})$ by using the laws of boolean algebra. From its truth table write it in minterm normal form. **[8a UPSC CSE 2018]**

Q4. Write the Boolean expression

$z(y+z)(x+y+z)$

in its simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form.

[5c UPSC CSE 2017]

Q5. $A\cdot(A+B+C)\cdot(\bar{A}+B+C)\cdot(A+\bar{B}+C)\cdot(A+B+\bar{C})$. Let A, B, C be Boolean variables, $\bar{A}$ denote complement of A, A+B is an expression for A OR B $A\cdot B$ is an expression for a AND B. Then simplify the following expression and draw a block diagram of the simplified expression, using AND and OR gates.

$A\cdot(A+B+C)\cdot(\bar{A}+B+C)\cdot(A+\bar{B}+C)\cdot(A+B+\bar{C})$. **[8c UPSC CSE 2016]**

Q6. Find the principal (or canonical disjunctive normal form in three variables p,q,r for the Boolean expression $((p \wedge q) \to r) \times ((p \wedge q) \to -r)$. Is the given Boolean expression a contradiction or a tautology? **[5c UPSC CSE 2015]**
Q7. use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$. **[5d UPSC CSE 2014]**
Q8. For any Boolean variables *x* and *y*, show that $x + xy = x$. **[8b UPSC CSE 2014]**
Q9. Let A be an arbitrary but fixed Boolean algebra with operation ^, ∨ and the zero and the unit element denoted by 0 and 1 respectively. Let x, y, z,....be elements of A. If $x, y \in A$ be such that $x \wedge y$ = 0 and $x \vee y$ = 1 then prove that $y = x'$. **[5d(ii) UPSC CSE 2011]**
Q10. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

[7b UPSC CSE 2011]
Q11. Suppose a computer spends 60 percent of its time handling a particular type of computation when running a given program and its manufactures make a change that improves its performance of that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change? **[5d(i) UPSC CSE 2010]**
Q12. If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. **[5d(ii) UPSC CSE 2010]**
Q13. Given the system of equations
$2x + 3y = 1$
$2x + 4y + z = 2$
$2y + 6z + Aw = 4$
$4z + Bw = C$
State the solvability and uniqueness conditions for the system. Give the solution when it exists.

[7a UPSC CSE 2010]
Q14. Using Boolean algebra, simplify the following expressions
(i) $a + a'b + a'b'c + a'b'c'd + ...$
(ii) $x'y'z + yz + xz$
where x' represents the complement of *x*. **[7c(iii) UPSC CSE 2010]**

CHAPTER 7. LOGARITHMS AND FLOWCHARTS

Q1. Write down an algorithm for Simpson's $\frac{1}{3}$ rule. Hence, compute $\int_0^1 x^2(1-x)\,dx$ correct up to three decimal places with step size $h=0.1$ and compare the result with its exact value.
[6b IFoS 2022]

Q2. Write down the algorithm and flowchart of Runge-Kutta method of fourth order to find the numerical solution at $x=0.8$ for
$\frac{dy}{dx}=\sqrt{2(x+y)},\ y(0.4)=0.82$.
Assume the step length $h=0.2$. **[7a IF Os 2021]**

Q1. Write down the algorithm and flowchart for solving numerically the differential equation $\frac{dy}{dx}=f(x,y)=1+x\cos y$ with initial condition: at $x=x_0, y=y_0$ and step length *h* by Euler's method up to $x=x_n=x_0=nh$. **[7a 2020 IFoS]**

Q2. Draw a flow chart and write a basic algorithm (in FORTRAN/C/C^{++}) for evaluating $y=\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule. **[5e UPSC CSE 2019]**

Q3. State the Newton-Raphson iteration formula to compute a root of an equation $f(x)=0$ and hence write a program in BASIC to compute a root of the equation $\cos x - xe^x = 0$ lying between 0 and 1. Use DEF function to define $f(x)$ and $f'(x)$. **[8b 2019 IFoS]**

Q4. Write down the basic algorithm for solving the equation: $xe^x-1=0$ by bisection method, correct to 4 decimal places. **[5e UPSC CSE 2018]**

Q5. Write a program in BASIC to multiply two matrices (checking for consistency for multiplication is required). **[5c 2018 IFoS]**

Q6. Write a program in BASIC to implement trapezoidal rule to compute $\int_0^{10} e^{-x^2}dx$ with 10 subdivisions. **[6d 2018 IFoS]**

Q7. Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method. **[8b UPSC CSE 2017]**

Q8. Write a BASIC program to compute the multiplicative inverse of a non-singular square matrix.

[5b 2017 IFoS]

Q9. Develop an algorithm for Newton-Raphson method to solve $\phi(x)=0$ starting with initial iterate x_0, n be the number of iterations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $\phi'(x)$. **[5c 2016 IFoS]**

Q10. Write a BASIC program to compute the product of two matrices. **[8a 2015 IFoS]**
Q11. Draw flow chart for Simpson's one-third rule. **[7b UPSC CSE 2014]**
Q12. Write a program in BASIC to integrate
$\int_0^1 e^{-2x}\sin x dx$ by Simpson's $\frac{1}{3}$rd rule with 20 subintervals. **[5b 2014 IFoS]**
Q13. Write a BASIC program to sum the series $S = 1 + x + x^2 + ... + x^n$, for $n = 30, 60$ and 90 for the values of $x = 0.1(0.1)0.3$. **[6d 2014 IFoS]**
Q14. Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$ starting with initial iterate x_0, n be the number of interations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$. **[7a UPSC CSE 2013]**
Q15. Write an algorithm to find the inverse of a given non-singular diagonally dominant square matrix using Gauss-Jordan method. **[7b 2013 IFoS]**
Q16. Draw a flow chart for testing whether a given real number is a prime or not. **[8c 2013 IFoS]**
Q17. Provide a computer algorithm to solve an ordinary differential equitation $\frac{dy}{dx} = f(x, y)$ in the interval $[a,b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$ using Euler's method. **[5c UPSC CSE 2012]**
Q18. Write a computer program to implement trapezoidal rule to evaluate
$\int_0^{10}\left(1 - e^{-\frac{x}{2}}\right)dx$. **[5c 2012 IFoS]**
Q19. Draw a flow chart for interpolation using Newton's forward difference formula.

[7c 2012 IFoS]

Q20. In a certain examination, a candidate has to appear for one major and two minor subjects. The rules for declaration of results are: marks for major are denoted by M_1 and for minors by M_2 and M_3. If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in any one of the two minors the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above.

[7c UPSC CSE 2012]

Q21. Draw a flow chart for Lagrange's interpolation formula. **[7c UPSC CSE 2011]**

Q22. Draw a flow chart to solve a quadratic equation with non-zero coefficients. The roots be classified as real distinct, real repeated and complex. **[8b 2011 IFoS]**

Q23. Draw a flow chart to declare the results for the following examination system:

60 candidates take the examination. Each candidate writes one major and two minor papers. A candidate is declared to have passed in the examination if he/she gets a minimum of 40 in all the three papers separately and an average of 50 in all the three papers put together. Remaining candidates fail in the examination with an exemption in major if they obtain 60 and above and exemption in each minor if they obtain 50 and more in that minor. **[6b 2011 IFoS]**

Q24. Draw a flow chart for finding the roots of the quadratic equitation $ax^2 + bx + c = 0$.

[6b 2010 IFoS]

LINEAR PROGRAMMING PROBLEMS (LPP)

1. FORMULATION OF LPP
- **GRAPHICAL SOLUTION**

2. SIMPLEX METHOD
- **BASIC FEASIBLE SOLUTIONS (BFS)**
- **SIMPLEX METHOD- BIG M METHOD**

3. DUALITY PRINCIPLE

4. TRANSPORTATION PROBLEM

5. ASSIGNMENT PROBLEM
- **TRAVELLING SALESMAN PROBLEM**

CHAPTER 1. FORMULATION OF LPP

Q1. An automobile dealer wishes to put four repairmen R_1, R_2, R_3 and R_4 to four different jobs J_1, J_2, J_3 and J_4. But R_3 cannot do the job J_2. The dealer has estimated the number of man-hours that would be required for each job-man on one-one basis as given in the following table:

	R_1	R_2	R_3	R_4
J_1	6	2	3	4
J_2	9	7	-	5
J_3	6	4	7	5
J_4	6	8	8	9

Formulate the above as a Linear Programming Problem. **[1d IFoS 2022]**

Q1. UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows:

Curtain length (in feet)	Number required
5	700
9	400
7	300

The width of all curtains is same as that of variable pieces. Form a linear programming problem in standard form that decides the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. **[1e UPSC CSE 2020]**

Q2. A company manufacturing air-coolers has two plants located at Bengaluru and Mumbai with a weekly capacity of 200 units and 100 units respectively. The company supplies air-coolers to its 4 showrooms situated at Mangalore, Bengaluru, Delhi and Goa which have a demand of 75, 100, 100 and 25 units respectively. Due to the difference in local taxes, showrooms charges, transportation cost and others, the profits differ. The profit (in Rs.) are shown in the following table:

From	To			
	Mangalore	Bengaluru	Delhi	Goa
Bengaluru	90	90	100	100
Mumbai	50	70	130	85

Plan the production program so as to maximize the profit. The company may have its production capacity at both plants partially or wholly unused. **[4d 2016 IFoS]**

Q3. An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3:3:4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2:4:2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. **[1e UPSC CSE 2018]**

Q4. A paint factory produces both interior and exterior paint from two raw materials M_1 and M_2. The basic data is as follows:

	Tons of raw material per tons		Maximum
	Exterior paint	Interior paint	Daily availability
Raw Material M1	6	4	24
Raw Material2	1	2	6
Profit per ton (Rs. 1,000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problem for this situation. **[1f UPSC CSE 2009]**

CHAPTER 2. GRAPHICAL SOLUTION

Q1. Prove that the linear programming problem

Maximize

$z = 3x_1 + 2x_2$

subject to the constraints:

$2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

does not admit an optimum basic feasible solution**. [1d IFoS 2022]**

Q1. Solve graphically the following LLP:

Max $z = 5x_1 - 3x_2$

subject to

$3x_1 + 2x_2 \le 12$

$-x_1 + x_2 \ge 1$

$-x_1 + x_2 \le 2$

$x_1, x_2 \geq 0$

If the objective function z is changed to Max $z = 6x_1 + 4x_2$, while the constraints remain the same, then comment on the number of solutions. Will $(4,0)$ be also a solution? **[1d 2020 IFoS]**

Q2. A firm manufactures two product A and B on which the profits earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2, while B requires one minute on M1 and one minute on M2. machine M1 is available for not more than 7 hours 30 minutes, while machine M2 is available for 10 hours during any working day. Find the number of units of products A and B to be manufactured to get maximum profit, using graphical method. **[1e 2019 IFoS]**

Q3. Using graphical method, find the maximum value of

$2x + y$ subject to

$4x + 3y \leq 12$

$4x + y \leq 8$

$4x - y \leq 8$

$x, y \geq 0$. **[1e UPSC CSE 2017]**

Q4. Find the maximum value of $5x + 2y$ with constraints $x + 2y \geq 1, 2x + y \leq 1, x \geq 0$ and $y \geq 0$ by graphical method. **[1e UPSC CSE 2016]**

Q5. Solve graphically:

Maximize $z = 7x + 4y$

subject to $2x + y \leq 2, x + 10y \leq 10$ and $x \leq 8$. (Draw your own graph without graph paper).

[1d 2015 IFoS]

Q6. Solve graphically:

Maximize $Z = 6x_1 + 5x_2$

subject to $2x_1 + x_2 \leq 16$

$x_1 + x_2 \leq 11$

$x_1 + 2x_2 \geq 6$

$5x_1 + 6x_2 \leq 90$

$x_1, x_2 \geq 0$. **[1e UPSC CSE 2014]**

Q7. Write the dual of the linear programming problem (LPP):

Minimize $Z = 18x_1 + 9x_2 + 10x_3$

subject to

$x_1 + x_2 + 2x_3 \geq 30$

$2x_1 + x_2 \geq 15$

$x_1, x_2, x_3 \geq 0$

Solve the dual graphically. Hence obtain the minimum objective function value of the above LPP.

[1e 2011 IFoS]

Q8. For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each in order to maximize his marks? **[1d UPSC CSE 2012]**

Q9. ABC Electricals manufactures and sells two models of lamps, L_1 and L_2, the profit per unit being Rs. 50 and Rs. 30, respectively. The process involves two workers W_1 and W_2, who are available for 40 hours and 30 hours per week, respectively. W_1 assembles each unit of L_1 in 30 minutes and that of L_2 in 40 minutes. W_2 paints each unit of L_1 in 30 minutes and that of L_2 in 15 minutes. Assuming that all lamps made can be sold, determine the weekly production figures that maximize the profit.

[4c 2010 IFoS]

CHAPTER3. BASIC FEASIBLE SOLUTIONS (BFS)

Q1. How many basic solutions are there in the following linearly independent set of equations? Find all of them.

$2x_1 - x_2 + 3x_3 + x_4 = 6$

$4x_1 - 2x_2 - x_3 + 2x_4 = 10$. **[3c UPSC CSE 2018]**

Q2. Prove that the set of all feasible solutions of a Linear Programming problem is a convex set.

[1e 2016 IFoS]

Q3. Consider the following linear programming problem:

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

subject to $x_1 + x_2 + 2x_3 + 3x_4 = 12$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

Using the definition, find its all *basic solutions*. Which of these are degenerate basic feasible solutions and which are non-degenerate basic feasible solutions?

Without solving the problem, show that it has an optimal solution. Which of the basic feasible solution(s) is/are optimal? **[3c UPSC CSE 2015]**

Q4. $x_1 = 4, x_2 = 1, x_3 = 3$ is a feasible solution of the system of equations

$2x_1 - 3x_2 + x_3 = 8$

$x_1 + 2x_2 + 3x_3 = 15$

Reduce the feasible solution to two different basic feasible solutions. **[4c 2013 IFoS]**

Q5. Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ for the linear programming problem

Maximize $Z = x_1 + 2x_2 + 3x_3$

subject to

$x_1 - x_2 + 3x_3 = 4$

$2x_1 + x_2 + x_3 = 6$

$x_1, x_2, x_3 \geq 0$. **[3c 2011 IFoS]**

SIMPLEX METHOD

Q1.

$2x_1 + x_2 + x_3 \leq 2$

$4x_1 + 2x_2 + x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

Maximize $Z = x_1 + x_2 + x_3$.

subject to

$2x_1 + x_2 + x_3 \leq 2$

$4x_1 + 2x_2 + x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

[3c UPSC CSE 2022]

Q1. Solve the linear programming problem using simplex method:

Minimize $z = -6x_1 - 2x_2 - 5x_3$

subject to $2x_1 - 3x_2 + x_3 \le 14$

$-4x_1 + 4x_2 + 10x_3 \le 46$

$2x_1 + 2x_2 - 4x_3 \le 37$

$x_1 \ge 2, x_2 \ge 1, x_3 \ge 3$. **[3b UPSC CSE 2020]**

Q2. Solve the following LPP by simplex method:

Max $z = 2x_1 + x_2$

subject to

$2x_1 - 2x_2 \le 1$

$2x_1 - 4x_2 \le 3$

$2x_1 + x_2 \le 2$

$x_1, x_2 \ge 0$

Dies there exist an alternate optimal solution? If yes, give one and hence find all the optimal solutions.

[3c 2020 IFoS]

Q3. Use simplex method to solve the following problem:

Maximize $z = 2x_1 + 5x_2$

subject to $x_1 + 4x_2 \le 24$

$3x_1 + x_2 \le 21$

$x_1 + x_2 \le 9$

$x_1, x_2 \ge 0$. **[3c 2019 IFoS]**

Q4. Solve by simplex method the following Linear Programming Problem:

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to the constraints

$x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$. **[1d 2018 IFoS]**

Q5. Solve the following linear programming problem by simplex method

Maximize

$z = 3x_1 + 5x_2 + 4x_3$

subject to

$2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

$x_1, x_2, x_3 \geq 0$. **[3c UPSC CSE 2017]**

Q6. Solve by simplex method the following LPP:
Minimize $Z = x_1 - 3x_2 + 2x_3$
subject to the constraints
$3x_1 - x_2 + 2x_3 \leq 7$
$-2x_1 + 4x_2 \leq 12$
$-4x_1 + 3x_2 + 8x_3 \leq 0$
and $x_1, x_2, x_3 \geq 0$. **[1d 2017 IFoS]**

Q7. Maximize
$z = 2x_1 + 3x_2 + 6x_3$
subject to
$2x_1 + x_2 + x_3 \leq 5$
$3x_2 + 2x_3 \leq 6$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$
Is the optimal solution unique? Justify your answer. **[2c UPSC CSE 2016]**

Q8. A manufacturer wants to maximize his daily output of bulbs which are made by two processes P_1 and P_2. If x_1 is the output by process P_1 and x_2 is the output by process P_2, then the total labour hours is given by $2x_1 + 3x_2$ and this cannot exceed 130, the total machine time is given by $3x_1 + 8x_2$ which cannot exceed 300 and the total raw material is given by $4x_1 + 2x_2$ and this cannot exceed 140. What should x_1 and x_2 be so that the total output $x_1 + x_2$ is maximum? Solve by the simplex method only.

[3c 2015 IFoS]

Q9. Find all optimal solutions of the following linear programming problem by the simplex method:
Maximize $Z = 30x_1 + 24x_2$
subject to
$5x_1 + 4x_2 \leq 200$
$x_1 \leq 32$
$x_2 \leq 40$
$x_1, x_2 \geq 0$. **[4c UPSC CSE 2014]**

Q10. Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$ subject to the constraints
$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$
$2x_1 - x_2 + x_3 + 2x_4 \leq 8$
$4x_1 - 2x_2 + x_3 - x_4 \leq 10$

$x_i \geq 0$. **[4c UPSC CSE 2013]**

Q11. Solve the following problem by Simplex Method. How does the optimal table indicate that the optimal solution obtained is not unique?
Maximize $z = 8x_1 + 7x_2 - 2x_3$ subject to the constraints
$x_1 + 2x_2 + 2x_3 \leq 12$
$2x_1 + x_2 - 2x_3 \leq 12$
$x_1, x_2, x_3 \geq 0$. **[2c 2012 IFoS]**

Q12. Solve by Simplex method, the following LP Problem:
Maximize, $Z = 5x_1 + 3x_2$
Constraints, $3x_1 + 5x_2 \leq 15$
$5x_1 + 2x_2 \leq 10$
$x_1, x_2 \geq 0$. **[1d UPSC CSE 2011]**

Q13. Solve the following linear programming problem by the simplex method:
Maximize $Z = 3x_1 + 4x_2 + x_3$
subject to
$x_1 + 2x_2 + 7x_3 \leq 8$
$x_1 + x_2 - 2x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$. **[3c 2010 IFoS]**

Q14. Maximize: $Z = 3x_1 + 5x_2 + 4x_3$
Subject to:
$2x_1 + 3x_2 \leq 8$
$3x_1 + 2x_2 + 4x_3 \leq 15$,
$2x_2 + 5x_3 \leq 10$,
$x_i \geq 0$. **[4b UPSC CSE 2009]**

SIMPLEX TWO PHSAE METHOD

Q1. Use two-phase method to solve the following linear programming problem:
Minimize $Z = x_1 + x_2$
subject to
$2x_1 + x_2 \geq 4$
$x_1 + 7x_2 \geq 7$
$x_1, x_2 \geq 0$. **[1e UPSC CSE 2022]**

SIMPLEX METHOD – BIG M METHOD

Q1. Solve the following linear programming problem using Big M method:

Maximize $Z = 4x_1 + 5x_2 + 2x_3$

Subject to

$2x_1 + x_2 + x_3 \geq 10$

$x_1 + 3x_2 + x_3 \leq 12$

$x_1 + x_2 + x_3 = 6$

$x_1, x_2, x_3 \geq 0$. **[4c UPSC CSE 2021]**

Q1. Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$x_1 + 2x_2 - 3x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + 2x_4 = 4$

and $x_1, x_2, x_3, x_4 \geq 0$. **[3b UPSC CSE 2019]**

Q2. Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function:

Minimize $z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$,

$x_1, x_2 \geq 0$. **[2b UPSC CSE 2018]**

Q3. Maximize $z = 2x_1 + 3x_2 - 5x_3$

subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0$. **[1e UPSC CSE 2013]**

6. DUALITY PRINCIPLE

Q1.Employ duality to solve the following linear programming problem:

Maximize

$z = 2x_1 + x_2$

subject to the constraints:

$x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0$. **[3c IFoS 2022]**

Q2. Consider the following Linear Programming Problem as primal:
Minimize $z = 30x_1 + 20x_2$
s/t, $3x_1 + 5x_2 \geq 100$
$2x_1 + x_2 \geq 120$
$5x_1 + 3x_2 \geq 90$
$x_1, x_2 \geq 0$
Then using the principle of duality, find the optimal solution of the primal. **[3c IFoS 2021]**

Q3. convert the following LPP into dual LPP:
Minimize $Z = x_1 - 3x_2 - 2x_3$
Subject to
$3x_1 - x_2 + 2x_3 \leq 7$
$2x_1 - 4x_2 \geq 12$
$-4x_1 + 3x_2 + 8x_3 = 10$
where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign. **[3c UPSC CSE 2021]**

Q1. Use graphical method to solve the linear programming problem.
Maximize $Z = 3x_1 + 2x_2$
subject to
$x_1 - x_2 \geq 1$
$x_1 + x_3 \geq 3$
and $x_1, x_2, x_3 \geq 0$. **[1e UPSC CSE 2019]**

Q2. Consider the following LPP,
Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$
subject to
$x_1 + x_2 + x_3 = 4$
$x_1 + 4x_2 + x_4 = 8$
and $x_1, x_2, x_3, x_4 \geq 0$
Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. **[4d UPSC CSE 2019]**

Q3. Solve the following linear programming problem by the Simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

Maximize $Z = 2x_1 - 4x_2 + 5x_3$
subject to
$x_1 + 4x_2 - 2x_3 \leq 2$
$-x_1 + 2x_2 + 3x_3 \leq 1$
$x_1, x_2, x_3 \geq 0$. **[4c UPSC CSE 2015]**

Q4. Solve the following LPP graphically:
Maximize $Z = 3x_1 + 4x_2$
subject to $x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 2$
$x_2 \leq 4$
$x_1, x_2 \geq 0$
Write the dual problem of the above and obtain the optimal value of the objective function of the dual without actually solving it. **[4c 2014 IFoS]**

Q5. Write down the dual of the following LP problem and hence solve it by graphical method:
Minimize, $Z = 6x_1 + 4x_2$
Constraints, $2x_1 + x_2 \geq 1$
$3x_1 + 4x_2 \geq 1.5$
$x_1, x_2 \geq 0$. **[4c UPSC CSE 2011]**

Q6. Construct the dual of the primal problem:
Maximize $z = 2x_1 + x_2 + x_3$, subject to the constraints $x_1 + x_2 + x_3 \geq 6, 3x_1 - 2x_2 + 3x_3 = 3$, $-4x_1 + 3x_2 - 6x_3 = 1$, and $x_1, x_2, x_3 \geq 0$. **[4c UPSC CSE 2010]**

Q7. Find the dual of the following linear programming problem:
Maximum $Z = 2x_1 - x_2 + x_3$
such that $x_1 + x_2 - 3x_3 \leq 8$
$4x_1 - x_2 + x_3 = 2$
$2x_1 + 3x_2 - x_3 \geq 5$
$x_1, x_2, x_3 \geq 0$. **[1f UPSC CSE 2008]**

CHAPTER 4. TRANSPORTATION PROBLEM

Q1.Find the initial basic feasible solution to the following transportation problem by the North-West corner rule and then optimize it.

	To			Availability
From	7	3	4	2
	2	1	3	3
	3	4	6	5
Demand	4	1	5	10

[4c IFoS 2022]

Q2. Starting with Least Cost Method, find all the solutions to the following transportation problem:

Warehouses

		I	II	III	IV	Supply
Plants	A	8	6	5	3	18
	B	6	7	6	8	20
	C	10	8	4	5	18
	Demand	15	16	12	13	

[4c IFoS 2021]

Q1. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem.

Sources	D_1	D_2	D_3	D_4	Availability
S_1	10	0	20	11	15
S_2	12	8	9	20	25
S_3	0	14	16	18	10
Demand	5	20	15	10	

[4c UPSC CSE 2020]

Q2. Find the minimum transportation cost using Vogel's approximation method for the following transportation problem:

Destinations

		D_1	D_2	D_3	D_4	Availability
Sources	S_1	9	16	15	9	15
	S_2	2	1	3	5	25
	S_3	6	4	7	3	20
	Demand	10	15	25	10	

[4c 2020 IFoS]

Q3. The capacities of three production facilities S1, S2 and S3 and the requirements of four destinations D1, D2, D3 and D4 and transportation costs in rupees are given in the following table:

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Find the minimum transportation cost using Vogel's Approximation Method (VAM).

[4d 2018 IFoS]

Q4. Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost.

Destinations

		D_1	D_2	D_3	D_4	D_5	Supply
Origins	O_1	4	7	0	3	6	14
	O_2	1	2	–3	3	8	9
	O_3	3	–1	4	0	5	17
	Demand	8	3	8	13	8	

[4b UPSC CSE 2017]

Q5. Solve the following transportation problem:

	D_1	D_2	D_3	Supply
O_1	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	60

[4c 2015 IFoS]

Q6. Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost:

		Destinations				
		D_1	D_2	D_3	D_4	Supply
	O_1	6	4	1	5	15

Origins	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
	Demand	6	10	15	4	

[2c UPSC CSE 2014]

Q7. Obtain the initial basic feasible solution for the transportation problem by North-West corner rule:

		Retail Shop					
		R_1	R_2	R_3	R_4	R_5	Supply
Factory	F_1	1	9	13	36	51	50
	F_2	24	12	16	20	1	100
	F_3	14	35	1	23	26	150
		100	70	50	40	40	

[1d 2014 IFoS]

Q8. By the method of Vogel, determine an initial basic feasible solution for the following transportation problem:

Product P_1, P_2, P_3 and P_4 have to be sent to destinations D_1, D_2 and D_3. The cost of sending product Pi to destinations D_j is C_{ij}, where the matrix

$$\left[C_{ij}\right]=\begin{bmatrix}10 & 0 & 15 & 5\\ 7 & 3 & 6 & 15\\ 0 & 11 & 9 & 13\end{bmatrix}$$

The total requirements of destinations D_1, D_2 and D_3 are given by 45, 45, 95 respectively and the availability of the products P_1, P_2, P_3 and P_4 are respectively 25, 35, 55 and 70.

[4c UPSC CSE 2012]

Q9. Find the initial basic feasible solution of the following minimum cost transportation problem by Least Cost (Matrix Minima) Method and using it find the optimal transpiration cost: -

		Destinations				
		D_1	D_2	D_3	D_4	Supply
Sources	S_1	5	11	12	13	10
	S_2	8	12	7	8	30
	S_3	12	7	15	6	35
Requirement		15	15	20	25	

[4c 2012 IFoS]

Q10. A steel company has three open-hearth furnaces and four rolling mills. Transportation costs (rupees per quintal) for shipping steel from furnaces to rolling mills are given in the following table:

	M_1	M_2	M_3	M_4	Supply

					(quintals)
F_1	29	40	60	20	7
F_2	80	40	50	70	10
F_3	50	18	80	30	18
Demand (quintals)					

Find the optimal shipping schedule. **[4c 2011 IFoS]**

Q11. Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories F_1, F_2, F_3 to five warehouses W_1, W_2, W_3, W_4, W_5 is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at F_1, F_2, F_3 is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:

	W_1	W_2	W_3	W_4	W_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. **[4c UPSC CSE 2010]**

Q12. Solve the following transportation problem:

		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
	Demand	30	50	20	40	30	10	180

by finding the initial solution by Matrix Minima Method. **[4c UPSC CSE 2008]**

CHAPTER 5. ASSIGNMENT PROBLEM

Q1. A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effective matrix. Assign all the jobs to these five employees to minimize the total processing time:

Employees

Jobs						
	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

[1e UPSC CSE 2021]

Q1.

		Machine				
		M_1	M_2	M_3	M_4	M_5
Operator	O_1	24	29	18	32	19
	O_2	17	26	34	22	21
	O_3	27	16	28	17	25
	O_4	22	18	28	30	24
	O_5	28	16	31	24	27

In a factor there are five operators O_1, O_2, O_3, O_4, O_5 and five machines M_1, M_2, M_3, M_4, M_5. The operating costs are given when the O_i operator operates the M_j machine $(i, j = 1, 2, ..., 5)$. But there is a restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate the fifth machine M_5. The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. **[4c UPSC CSE 2018]**

Q2. A computer centre has four expert programmers. The centre needs four application to programs to be developed. The head of the centre after studying carefully the programs to be developed, estimates the computer times in hours required by the experts to the application programs as follows:

		Programs			
		A	B	C	D
Programmer	P_1	5	3	2	8
	P_2	7	9	2	6
	P_3	6	4	5	7
	P_4	5	7	7	8

Assign the programs to the programmers in such a way that total computer time is least.

[4d 2017 IFoS]

Q3. Solve the minimum time assignment problem:

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	3	12	5	14
	J_2	7	9	8	12
	J_3	5	11	10	12
	J_4	6	14	4	11

[4a UPSC CSE 2013]

Q4. Find the optimal assignment cost from the following cost matrix:

	A	B	C	D
I	4	5	4	3
II	3	2	2	6
III	4	5	3	5
IV	2	4	2	6

[1d 2013 IFoS]

Q5. A captain of a cricket team has to allot four middle-order batting positions to four batsmen. The average number of runs scored by each batsman at these positions are as follows. Assign each batsman his batting position for *maximum* performance:

Batting	IV	V	VI	VII
Batsman				
A	40	25	20	35
B	36	30	24	40
C	38	30	18	40
D	40	23	15	33

[1e 2010 IFoS]

TRAVELLING SALESMAN PROBLEM

Q1. A salesman wants to visit cities C1, C2, C3 and C4. He does not want to visit any city twice before completing the four of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given below in the table. Find the least cost route.

		To City			
		C_1	C_2	C_3	C_4
	C_1	0	30	80	50

From City	C_2	40	0	140	30
	C_3	40	50	0	20
	C_4	70	80	130	0

[4c 2019 IFoS]

Q2. Solve the following assignment problem to maximize the sales:

		Territories				
		I	II	III	IV	V
Salesmen	A	3	4	5	6	7
	B	4	15	13	7	6
	C	6	13	12	5	11
	D	7	12	15	8	5
	E	8	13	10	6	9

[1e UPSC CSE 2015]

Q3. Solve the following Salesman problem:

	A	B	C	D
A	∞	12	10	15
B	16	∞	11	13
C	17	18	∞	20
D	13	11	18	∞

[2c 2013 IFoS]

FLUID DYNAMICS

1. KINEMATICS OF FLUID
2. EULER'S EQUATION OF MOTION
3. MOTION IN 2D- SOURCES & SINKS
4. AXISYMMETRIC MOTION
5. VORTEX MOTION
6. NAVIER STOKES EQUATION

CHAPTER 1.KINEMATICS OF FLUID

Q1. If the velocity of an incompressible fluid at the point (x, y, z) is given by $(-Ay, Ax, 0)$, then prove that the surfaces intersecting the stream lines orthogonally exist and are the planes through z-axis, although the velocity potential does not exist. Discuss the nature of the fluid flow.
[6c IFoS 2022]

Q2. The velocity components of an incompressible fluid in spherical polar coordinates (r, θ, ψ) are $\left(2Mr^{-3}\cos\theta, Mr^{-2}\sin\theta, 0\right)$, where M is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of the streamlines. **[5e UPSC CSE 2022]**

Q3. Verify whether the motion given by $\vec{q} = \left(3x\hat{i} - 2y\hat{j}\right)xy^2$ is a possible fluid motion. If so, is it of the potential kind? Accordingly find out the streamlines and the velocity potential or the angular velocity if the fluid was replaced by a rigid solid. **[6c IFoS 2021]**

Q4. Show that $\vec{q} = \dfrac{\lambda\left(-y\hat{i} + x\hat{j}\right)}{x^2 + y^2}$, ($\lambda =$constant) is a possible incompressible fluid motion. Determine the streamlines. Is the kind of the motion potential? If yes, then find the velocity potential.**[7c UPSC CSE 2021]**

Q1. A velocity potential in a two-dimensional fluid flow is given by $\phi(x, y) = xy + x^2 - y^2$. Find the stream function for this flow. **[7c UPSC CSE 2020]**

Q2. In a fluid flow, the velocity vector is given by $\vec{V} = 2x\vec{i} + 3y\vec{j} - 5z\vec{k}$. Determine the equation of the streamline passing through a point $A = (4, 9, 1)$. **[6c 2020 IFoS]**

Q3. Consider the flow field given by $\psi = a\left(x^2 - y^2\right)$, 'a' being a constant. Show that the flow is irrigational. Determine the velocity potential for this flow and show that the streamlines and equivelocity potential curves are orthogonal. **[5d 2019 IFoS]**

Q4. Consider that the region $0 \le z \le h$ between the planes $z = 0$ and $z = h$ is filled with viscous incompressible fluid. The plane $z = 0$ is held at rest and the plane $z = h$ moves with constant velocity $V\hat{j}$. When conditions are steady, assuming there is no slip between the fluid and either boundary, and neglecting body forces, show that the velocity profile between the plates is parabolic. Find the tangential stress at any point $P(x, y, z)$ of the fluid and determine the drag per unit area on both the planes. **[8a 2019 IFoS]**

Q5. For an incompressible fluid flow, two components of velocity (u,v,w) are given by $u = x^2 + 2y^2 + 3z^2$, $v = x^2y - y^2z + zx$. Determine the third component w so that they satisfy the equation of continuity. Also, find the z-component of acceleration. **[(5c) UPSC CSE 2018]**

Q6. For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}\left(x^3 - y^3\right)$.Determine the velocity components along the directions x and y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not.

[8b UPSC CSE 2018]

Q7. If the velocity of an incompressible fluid at the point (x,y,z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2$$

then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further, determine the streamlines. **[8c UPSC CSE 2017]**

Q8. A stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to be steady and diverging from the vertex of the cone, then prove that

$$\frac{u}{V} = \frac{D^2}{d^2} e^{\left(u^2 - V^2\right)/2K}$$

where K is the pressure divided by the density and is constant. **[7c UPSC CSE 2017]**

Q9. Find the streamlines and pathlines of the two dimensional velocity field:

$u = \frac{x}{1+t}, v = y, w = 0$. **[8b 2017 IFoS]**

Q10. In a steady fluid flow, the velocity components are $u = 2kx, v = 2ky$ and $w = -4kz$. Find the equation of a streamline passing through $(1,0,1)$. **[(6c) 2015 IFoS]**

Q11. Suppose $\vec{v} = (x - 4y)\hat{i} + (4x - y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow. **[(8b) 2015 IFoS]**

Q12. Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}\right]$, determine the streamline.

[(7c) UPSC CSE 2014]

Q13. Find the condition that $f(x,y,\lambda) = 0$ should be a possible system of streamlines for steady irrotational motion in two dimensions, where λ is a variable parameter.

[5e 2014 IFoS]

Q14. Prove that

$$\frac{x^2}{a^2}\tan^2 t+\frac{y^2}{b^2}\cot^2 t=1$$

is a possible form for the bounding surface of a liquid and find the velocity components.

[8c 2014 IFoS]

Q15. Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the stream lines are

$$u,v,w=\mu\left(\frac{\partial\varphi}{\partial x},\frac{\partial\varphi}{\partial y},\frac{\partial\varphi}{\partial z}\right)$$

where μ and φ are functions of x,y,z,t. **[5d UPSC CSE 2013]**

Q16. Find the values of a and b in the 2-D velocity field $\vec{v}=\left(3y^2-ax^2\right)\hat{i}+bxy\hat{j}$ so that the flow becomes incompressible and irrational. Find the stream function of the flow. **[7a 2013 IFoS]**

Q17. Show that $\phi=xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q}\to 0$ as $r\to\infty$, find the surfaces of constant speed.

[8b UPSC CSE 2012]

Q18. Show that

$$u=\frac{A\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2},\ v=\frac{2Axy}{\left(x^2+y^2\right)^2},\ w=0$$

are components of a possible velocity vector for invisoid incompressible fluid flow. Determine the pressure associated with this velocity field. **[7a 2012 IFoS]**

Q19. Is

$$\vec{q}=\frac{k^2\left(x\hat{j}-y\hat{i}\right)}{x^2+y^2}$$

a possible velocity vector of an incompressible fluid motion? If so, find the stream function and velocity potential of the motion. **[8c 2011 IFoS]**

Q20. A two-dimensional flow field is given by $\psi=xy$. Show that -

(i) the flow is irrotational;

(ii) ψ and ϕ satisfy Laplace equation

Symbols ψ and ϕ convey the usual meaning. **[5e 2010 IFoS]**

Q21. Show that $\phi=(x-t)(y-t)$ represents the velocity potential of an incompressible two-dimensional fluid. Further show that the streamlines at time t are the curves

$(x-t)^2-(y-t)^2=$ constant. **[7b 2010 IFoS]**

CHAPTER 2. EULER'S EQUATION OF MOTION

Q1. A sphere of radius R, whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is Π, so that the pressure at the surface of the sphere at time *t* is $\Pi+\frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2}+\left(\frac{dR}{dt}\right)^2\right\}$. **[8b UPSC CSE 2019]**

Q2. Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and v be the velocity at a distance x from a fixed point at time *t*, then $\frac{\partial^2\rho}{\partial t^2}=\frac{\partial^2}{\partial x^2}\left\{\rho\left(v^2+k\right)\right\}$. **[5d UPSC CSE 2018]**

CHAPTER 3. MOTION IN 2D- SOURCES & SINK

Q1. Two sources of strength $\frac{m}{2}$ are placed at the point $(\pm a,0)$. Show that at any point on the circle $x^2+y^2=a^2$, the velocity is parallel to the *y*-axis and is inversely proportional to *y*.

[8c UPSC CSE 2020]

Q2. In a two-dimensional fluid flow, the velocity components are given by $u=x-ay$ and $v=-ax-y$, where a is constant. Show that the velocity potential exists for this flow and determine the appropriate velocity potential. Also, determine the corresponding stream function that would represent the flow. **[7b 2020 IFoS]**

Q3. Two sources, each of strength *m*, are placed at the point $(-a,0)$, $(a,0)$ and a sink of strength $2m$ at origin. Show that the stream lines are the curves $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2+\lambda xy\right)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $\left(2ma^2\right)/r_1r_2r_3$, where r_1,r_2 and r_3 are the distances of the points from the sources and the sink, respectively. **[8c UPSC CSE 2019]**

Q4. In the case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u in a fixed direction, where u is a variable. Show that the maximum value of the velocity at any point of the fluid is $2u$. Prove that the force necessary to hold the disc is $2m\dot{u}$, where *m* is the mass of the liquid displaced by the disc. **[7d 2018 IFoS]**

Q5. Two sources, each of strength *m*, are placed at the points $(-a,0),(a,0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curves $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2+\lambda xy\right)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $\left(2ma^2\right)/r_1r_2r_3$, where r_1,r_2,r_3 are the distances of the point from the sources and the sink. **[8d 2018 IFoS]**

Q6. A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{i}$ .Show that the velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\theta$, where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{i}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2\sin^2\theta - 2m\cos\theta =$ constant. **[6b UPSC CSE 2016]**

Q7. Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points.

[5d UPSC CSE 2015]

Q8. If fluid fills the region of space on the positive side of the x-axis, which is a rigid boundary and if there be a source m at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{\{2ab(a+b)\}}$ where ρ is the density of the fluid. **[8b UPSC CSE 2013]**

Q9. With usual notations, show that ϕ and ψ for a uniform flow past a stationary cylinder are given by

$$\phi = U\cos\theta\left(r + \frac{a^2}{r}\right)$$

$$\psi = U\sin\theta\left(r - \frac{a^2}{r}\right).$$ **[5e 2011 IFoS]**

CHAPTER 4. AXISYMMETRIC MOTION

Q1. The space between two concentric spherical shells of radii $a, b(a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential

$$\phi = \frac{\left\{a^3U\left(1+\frac{1}{2}b^3r^{-3}\right)x - b^3V\left(1+\frac{1}{2}a^3r^{-3}\right)y\right\}}{\left(b^3 - a^3\right)}$$

where $r^2 = x^2 + y^2 + z^2$, the coordinates being rectangular. Evaluate the velocity at any point of the liquid. **[7b UPSC CSE 2016]**

Q2. A sphere is at rest in an infinite mass of homogenous liquid of density ρ, the pressure at infinity being P. If the radius R of the sphere varies in such a way that $R = a + b\cos nt$, where $b < a$, then find the pressure at the surface of the sphere at any time. **[8c 2016 IFoS]**
Q3. In an axisymmetric motion, show that stream function exists due to equation of continuity. Express the velocity components in terms of the stream function. Find the equation satisfied by the stream function if the flow is irrotational. **[8c UPSC CSE 2015]**
Q4. A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

[5e UPSC CSE 2012]

CHAPTER 5. VORTEX MOTION

Q1.Verify that $w = ik\log\left(\frac{z - ia}{z + ia}\right)$ is the complex potential of a steady fluid flow about a circular cylinder, the plane $y = 0$ being a rigid boundary. Further show that the fluid exerts a downward force of magnitude $\left(\frac{\pi\rho k^2}{2a}\right)$ per unit length on the cylinder, where ρ is the fluid density. **[7b IFoS 2022]**
Q2. Two point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-\frac{k}{2}$ is situated at the origin. Show that the fluid motion is stationary and also find the equations of streamlines. If the streamlines, which pass through the stagnation points, meet the x-axis at $(\pm b, 0)$, then show that $3\sqrt{3}\left(b^2 - a^2\right)^2 = 16a^3 b$. **[7c UPSC CSE 2022]**
Q3. Discuss the flow given by the complex potential
$w = \log_e\left(z - \frac{a^2}{z}\right)$.
Draw sketches of the streamlines and explain the flow directions along the streamlines. **[7b IFoS 2021]**

Q4. What arrangements of sources and sinks can have the velocity potential $w = \log_e\left(z - \frac{a^2}{z}\right)$? Draw the corresponding sketch of the streamlines and prove that two of them subdivide into the circle $r = a$ and the axis of y.

[5e UPSC CSE 2021]

Q1. The velocity vector in the flow field is given by
$\vec{q} = (az - by)\hat{i} + (bx - cz)\hat{j} + (cy - ax)\hat{k}$
where a, b, c are non-zero constants. Determine the equations of vortex lines.

[8c 2017 IFoS]

Q2. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r} \right]$ possess vorticity, where $\vec{q}(u, v, m)$ is the velocity in the Cartesian frame, $\vec{r} = (x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$? **[5b UPSC CSE 2016]**

Q3. Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + \nu \nabla^2 \vec{\Omega}$$

where $\vec{q}$ is the velocity vector with $\vec{\Omega} = \nabla \times \vec{q}$. **[5d 2014 IFoS]**

Q4. If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circular cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^3}{(n-1)K}$. Find the velocity at any point of the liquid. **[8c UPSC CSE 2013]**

Q5. Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + \nu \nabla^2 \vec{\Omega}.$$ **[5d 2012 IFoS]**

Q6. An infinite row of equidistant rectilinear vortices are at a distance a apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function. **[8b UPSC CSE 2011]**

Q7. In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solutions of Laplace's equation.

[5f UPSC CSE 2010]

Q8. When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distances from its axis, show that the path of each vortex is given by the equation $(r^2 \sin^2\theta - b^2)(r^2 - a^2) = 4a^2 b^2 r^2 \sin^2\theta$,

θ being measured from the line through centre perpendicular to the joint of the vortices.

[8b UPSC CSE 2010]

Q9. Show that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = \left(\vec{\Omega} \cdot \nabla\right)\vec{q} + v\nabla^2\vec{\Omega}$$

where v is the kinematic viscosity. **[8c 2010 IFoS]**

CHAPTER 6. NAVIER STOKES EQUATION

Q1. Find Navier-Stokes equation for a steady laminar flow of a viscous incompressible fluid between two infinite parallel plates. **[8c UPSC CSE 2014]**

Q2. For a steady Poiseuille flow through a tube of uniform circular cross-section, show that

$w(R) = \frac{1}{4}\left(\frac{p}{\mu}\right)\left(a^2 - R^2\right)$. **[7a UPSC CSE 2011]**

MECHANICS

1. MOMENT OF INERTIA
2. EQUATION OF MOTION IN 2D/ D'ALEMBERT PRINCIPLE
3. LAGRANGE'S EQUATION OF MOTION
4. HAMILTON'S EQUATION OF MOTION

CHAPTER 1. MOMENT OF INERTIA

Q1. Find the moment of inertia of a right circular solid cone about one of its slant sides (generator) in terms of its mass M, height *h* and the radius of base as *a*.

[6C UPSC CSE 2022]

Q1. Prove that the moment of inertia of a triangular lamina ABC about any axis through A in its plane is $\frac{M}{6}\left(\beta^2+\beta\gamma+\gamma^2\right)$

where M is the mass of the lamina and β,γ are respectively the length of perpendiculars from B and C on the axis. **[5e UPSC CSE 2020]**

Q2. Show that the moment of inertia of an elliptic area of mass M and semi-axis a and b about a semi-diameter of length *r* is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. Further, prove that the moment of inertia about a tangent is $\frac{5M}{4}p^2$, where *p* is the perpendicular distance from the centre of the ellipse to the tangent.

[5e UPSC CSE 2017]

Q3. A uniform rectangular parallelepiped of mass M has edges of lengths $2a, 2b, 2c$. Find the moment of inertia of this rectangular parallelepiped about the line through its centre parallel to the edge of length $2a$. **[5c 2017 IFoS]**

Q4. Calculate the moment of inertia of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(i) relative to the *x*-axis

(ii) relative to the *y*-axis and

(iii) relative to the origin. **[5e 2016 IFoS]**

Q5. Find the moment of inertia of a right solid cone of mass M, height *h* and radius of whose base is *a*, about its axis. **[8a 2016 IFoS]**

Q6. Calculate the moment of inertia of a solid uniform hemisphere $x^2+y^2+z^2=a^2, z\geq 0$ with mass *m* about the OZ-axis. **[5e UPSC CSE 2015]**

Q7. Find the moment of inertia of a uniform mass M of a square shape with each side a about its one of the diagonals. **[7b 2015 IFoS]**

Q8. Show that the moment of inertia of a uniform rectangular mass M and sides 2*a* and 2*b* about a diagonal is $\frac{2Ma^2b^2}{3\left(a^2+b^2\right)}$. **[6b 2014 IFoS]**

Q9. Four solid spheres A, B, C and D, each of mass *m* and radius *a*, are placed with their centres on the four corners of a square of side *b*. Calculate the moment of inertia of the system about a diagonal of the square. **[5e UPSC CSE 2013]**

Q10. A pendulum consists of a rod of length 2*a* and mass *m*; to one end of which a spherical bob of radius $a/3$ and mass 15 m is attached. Find the moment of inertia of the pendulum:

(i) about an axis through the other end of the rod and at right angles to the rod.

(ii) about a parallel axis through the centre of mass of the pendulum.
[Given: The centre of mass of the pendulum is $a/12$ above the centre of the sphere.]

[8a UPSC CSE 2012]

Q11. Let a be the radius of the base of a right circular cone of height h and mass M. Find the moment of inertia of that right circular cone about a line through the vertex perpendicular to the axis.

[5e UPSC CSE 2011]

Q12. From a uniform sphere of radius a, a spherical sector of vertical angle 2α is removed. Find the moment of inertia of the remainder mass M about the axis of symmetry. **[8a 2011 IFoS]**

Q13. A uniform lamina is bounded by a parabolic arc of latus rectum $4a$ and a double ordinate at a distance b from the vertex.

If $b=\frac{a}{3}\left(7+4\sqrt{7}\right)$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there. **[5e UPSC CSE 2010]**

Q14. Show that the sum of the moments of inertia of an elliptic area about any two tangents at right angles is always the same. **[5d 2010 IFoS]**

CHAPTER 2. EQUATION OF MOTION IN 2D / D'ALEMBERT PRINCIPLE

Q1. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α. Prove that the direction of reaction at the hinge makes with the vertical, an angle $\tan^{-1}\left[\frac{3}{4}\tan\alpha\right]$. **[1d IFoS 2022]**

Q2. A particle is constrained to move along a circle lying in the vertical xy-plane. With the help of the D'Alembert's principle, show that its equation of motion is $\ddot{x}y-\ddot{y}x-gx=0$, where g is the acceleration due to gravity. **[5d UPSC CSE 2021]**

Q1. A uniform rod OA, of length $2a$, free to turn about its end O, revolves with angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ; find the value of α.

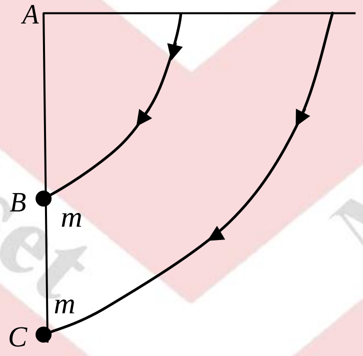
[5c UPSC CSE 2019]

Q2. A circular cylinder of radius a and radius of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane through axes moves in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$. **[6c UPSC CSE 2019]**

Q3. A uniform rod OA of length $2a$ is free to turn about its end O, revolves with uniform angular velocity ω about a vertical axis OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or

$\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. **[7c 2014 IFoS]**

Q4. A weightless rod ABC of length $2a$ is movable about the end A which is fixed and carries two particles of mass m each one attached to the mid-point B of the rod and the other attached to the end C of the rod. If the rod is held in the horizontal position and released from rest and allowed to move, show that the angular velocity of the rod when it is vertical is $\sqrt{\frac{6g}{5a}}$.



[8b 2012 IFoS]

Q5. The ends of a heavy rod of length $2a$ are rigidly attached to two light rings which can respectively slides on the thin smooth fixed horizontal and vertical wires O_x and O_y. The rod starts at an angle α to the horizon with an angular velocity $\sqrt{[3g(1-\sin\alpha)/2a]}$ and moves downwards. Show that it will strike the horizontal wire at the end of time

$-2\sqrt{a/(3g)}\log\left[\tan\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\cot\frac{\pi}{8}\right]$. **[8a UPSC CSE 2011]**

CHAPTER 3. LAGRANGE'S EQUATION OF MOTION

Q1. A particle at a distance r from the centre of force moves under the influence of the central force $F=-\frac{k}{r^2}$, where k is a constant. Obtain the Lagrangian and derive the equations of motion. **[5d UPSC CSE 2022]**

Q2. Derive the Lagrange's equation for a spherical problem. **[8a IFoS 2021]**

Q3. Obtain the Lagrangian equation for the motion of a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley. **[6C UPSC CSE 2021]**

Q1. A particle is attached to a center by a force which varies inversely as the cube of its distance from the center. Identify the generalized coordinates and write down the Lagrangian of the system. Derive then the equations of motion and solve them for the orbits. Discuss how the nature of orbits depends on the parameters of the system. **[8a 2020 IFoS]**

Q2. For a dynamical system

$$T=\frac{1}{2}\left\{(1+2k)\dot{\theta}^2+2\dot{\theta}\dot{\varphi}+\dot{\varphi}^2\right\},$$

$$V=\frac{n^2}{2}\left\{(1+k)\theta^2+\varphi^2\right\},$$

where θ, φ are coordinates and n, k are positive constants, write down the Lagrange's equations of motion and deduce that

$$\left(\ddot{\theta}-\ddot{\varphi}\right)+n^2\left(\frac{1+k}{k}\right)(\theta-\varphi)=0.$$

Further show that if $\theta=\varphi, \dot{\theta}=\dot{\varphi}$ at $t=0$, then $\theta=\varphi$ for all t. **[6c 2019 IFoS]**

Q3. Suppose the Lagrangian of a mechanical system is given by

$$L=\frac{1}{2}m\left(a\dot{x}^2+2b\dot{x}\dot{y}+c\dot{y}^2\right)-\frac{1}{2}k\left(ax^2+2bxy+cy^2\right),$$

where $a,b,c,m(>0),k(>0)$ are constants and $b^2\neq ac$. Write down the Lagrangian equations of motion and identify the system. **[6c UPSC CSE 2018]**

Q4. A particle of mass m is constrained to move on the inner surface of a cone of semi-angle α under the action of gravity. Write the equation of constraint and mention the generalized coordinates. Write down the equation of motion. **[8c 2018 IFoS]**

Q5. Two uniform AB, AC, each of mass m and length $2a$, are smoothly hinged together at A and move on horizontal plane. At time t, the mass centre of the rods is at the point (ξ,η) referred to fixed perpendicular axes O_x, O_y in the plane, and the rods make angles $\theta\pm\phi$ with O_x. Prove that the kinetic energy of the system is

$$m\left[\dot{\xi}^2+\dot{\eta}^2+\left(\frac{1}{3}+\sin^2\phi\right)a^2\dot{\theta}^2+\left(\frac{1}{3}+\cos^2\phi\right)a^2\dot{\phi}^2\right].$$

Also derive Lagrange's equations of motion for the system if an external force with components $[X,Y]$ along axes acts at A. **[6c UPSC CSE 2017]**

Q6. Consider a mass m on the end of a spring of natural length l and spring constant k. Let y be the vertical coordinate of the mass as measured from the top of the spring. Assume that the mass can only move up and down in the vertical direction. Show that

$$L=\frac{1}{2}my'^2-\frac{1}{2}k(y-l)^2+mgy$$

Also determine and solve the corresponding Euler-Lagrange equations of motion.

[8a 2017 IFoS]

Q7. A hoop with radius r is rolling, without slipping, down an inclined plane of length l and with angle of inclination ϕ. Assign appropriate generalized coordinates to the system. Determine the constraints, if any. Write down the Lagrangian equations for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane. **[8b UPSC CSE 2016]**

Q8. A bead slides on a wire in the shape of a cycloid described by the equations

$$x=a(\theta-\sin\theta)$$

$$y=a(1+\cos\theta)$$

where $0\le\theta\le2\pi$ and the friction between the bead and the wire is negligible. Deduce Lagrange's equation of motion. **[8b 2016 IFoS]**

Q9. Two equal rods AB and BC, each of length l, smoothly joined at B, are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where $n^2=\left(3\pm\frac{6}{\sqrt{7}}\right)\frac{g}{l}$. **[8a UPSC CSE 2013]**

Q10. Find the Lagrangian for a simple pendulum and obtain the equation describing its motion.

[5d 2011 IFoS]

CHAPTER 4. HAMILTON'S EQUATION OF MOTION

Q1. By writing down the Hamiltonian, find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2+y^2=R^2$, R is a constant. The particle is subject to a force directed towards the origin and proportional to the distance r of the particle from the origin given by $\vec{F}=-k\vec{r}$, k is a constant. **[6c UPSC CSE 2020]**

Q2. Find the condition on *a*, *b*, *c* (real numbers) such that the dynamical system with equations $\dot{p} = aq - q^2, \dot{q} = bp + cq$ is Hamiltonian. Compute also the Hamiltonian of the system.

[5d 2020 IFoS]

Q3. Using Hamilton's equation, find the acceleration for a sphere rolling down a rough inclined plane, if *x* be the distance of the point of contact of the sphere from a fixed point on the plane.

[7a UPSC CSE 2019]

Q4. Consider a mass-spring system consisting of a mass *m* and a linear spring of stiffness *k* hanging from a fixed point. Find the equation of motion using the Hamiltonian method, assuming that the displacement *x* is measured from the unscratched position of the string.

[7b 2019 IFoS]

Q5. The Hamiltonian of a mechanical system is given by,
$H = p_1 q_1 - a q_1^2 + b q_2^2 - p_2 q_2$, where a, b are the constants. Solve the Hamiltonian equations and show that $\frac{p_2 - b q_2}{q_1}$ = constant. **[7c UPSC CSE 2018]**

Q6. For a particle having charge *q* and moving in an electromagnetic field, the potential energy is $U = q\left(\phi - \vec{v} \cdot \vec{A}\right)$, where ϕ and $\vec{A}$ are, respectively, known as the scalar and vector potentials. Derive expression for Hamiltonian for the particle in the electromagnetic field.

[6c 2018 IFoS]

Q7. Consider a single free particle of mass *m*, moving in space under no forces. If the particle starts from the origin at $t = 0$ and reaches the position (x, y, z) at time τ, find the Hamilton's characteristic function S as a function of x, y, z, τ.
[5c UPSC CSE 2016]

Q8. Solve the plane pendulum problem using the Hamiltonian approach and show that H is a constant of motion. **[6b UPSC CSE 2015]**

Q9. A Hamiltonian of a system with one degree of freedom has the form

$$H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{b\alpha}{2} q^2 e^{-\alpha t}\left(\alpha + be^{-\alpha t}\right) + \frac{k}{2} q^2$$

where α, b, k are constants, *q* is the generalized coordinate and *p* is the corresponding generalized momentum.

(i) Find a Lagrangian corresponding to this Hamiltonian.

(ii) Find an equivalent Lagrangian that is not explicitly dependent on time.

[7c UPSC CSE 2015]

Q10. Derive the Hamiltonian and equation of motion for a simple pendulum.

[5c 2015 IFoS]

Q11. Find the equation of motion of a compound pendulum using Hamilton's equations.

[5e UPSC CSE 2014]

Q12. Derive the Hamiltonian and equation of motion for a simple pendulum. **[5c 2013 IFoS]**

Q13. Obtain the equations governing the motion of a spherical pendulum. **[5d UPSC CSE 2012]**

Q14. Derive the differential equation of motion for a spherical pendulum. **[6b 2012 IFoS]**

Q15. A sphere of radius *a* and mass *m* rolls down a rough plane inclined at an angle α to the horizontal. If *x* be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[8a UPSC CSE 2010]**

CHAPTER 5. Work & Energy (Equilibrium/Centre of Mass)

Q1. A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$$

where a is the length of the plank. **[8b 2014 IFoS]**

Work & Energy (Statics)

Q2. A mass m_1, having at the end of a string, draws a mass m_2 along the surface of a smooth table. If the mass on the table be doubled, the tension of the string is increased by one-half. Show that $m_1 : m_2 = 2:1$. **[(8a) 2010 IFoS]**

Mentor: Upendra Singh

Designation
Founder- Mindset Makers for UPSC
Guide at Insights IAS and Nirman IAS (Prev.)
Sr. Faculty at Physics By Fiziks (Higher Studies(Mathematics) since year 2013)
Chairman: Patiyayat Farmer Producer Company Limited.
Associated in Policy making for Rural upliftment/education with Govt. Uttar Pradesh.

Educational Background
Alumnus, IIT Delhi (2011 Batch, AIR49)
UPSC CSE Journey- On the basis of one of top scores multiple times given by UPSC, I'm confident about each stage of CSE. Preliminary (Developed special strategies KNOWLEDGE+STRATEGY), Mains_ Good command over Essay and GS4+GS2, optional mathematics), Personality(Interview) - Developed it through experiences and learnings through reading books from multiple dimensions during UPSC journey, Part of Interview board for Higher studies.

Message from Upendra Sir

HOW TO CHOOSE OPTIONAL FOR UPSC CSE (Main Exam)

(I had mathematics optional and one of my close friend took history optional)

Since years, I've been teaching higher mathematics, guiding UPSC aspirants too. So I guess, can answer you for the asked question.

What research needed to be done before choosing optional for UPSC(CSE)?

There are few parameters on which we need to have idea _

1- My own interest/ my core subject in UG or PG.

2- Resources/guidance/coaching availability for that subject.

3- Pattern of marks in previous years- For this, we should read some blogs written by selected persons with that optional. Advice- don't rely on many youTube videos, those may be overhyped.

4- Duration to cover the syllabus- There are some optionals which take less time to cover. example- Hindi literature takes less time to finish as compared to history . But here we must aware about scores low and high frequently.

5- Management- Let's say if you choose mathematics, obviously it'll have no link with general studies. You need a different mindset to prepare Mathematics Optional. You need lot of practice. It may exhaust you and you can compromise with your other subjects. The most important is - in main exam hectic schedule. Mathematics may be a reason of anxiety and restlessness. That 5 days gap seems long but during mains preparation we're exhausted and literally tired to grasp mathematics revision. So untill unless, you don't have good background and interest in mathematics, you should avoid it. Solution- during preparation , you have to develop traits and qualities to handle/manage it. You can talk to me at the mentoring number.

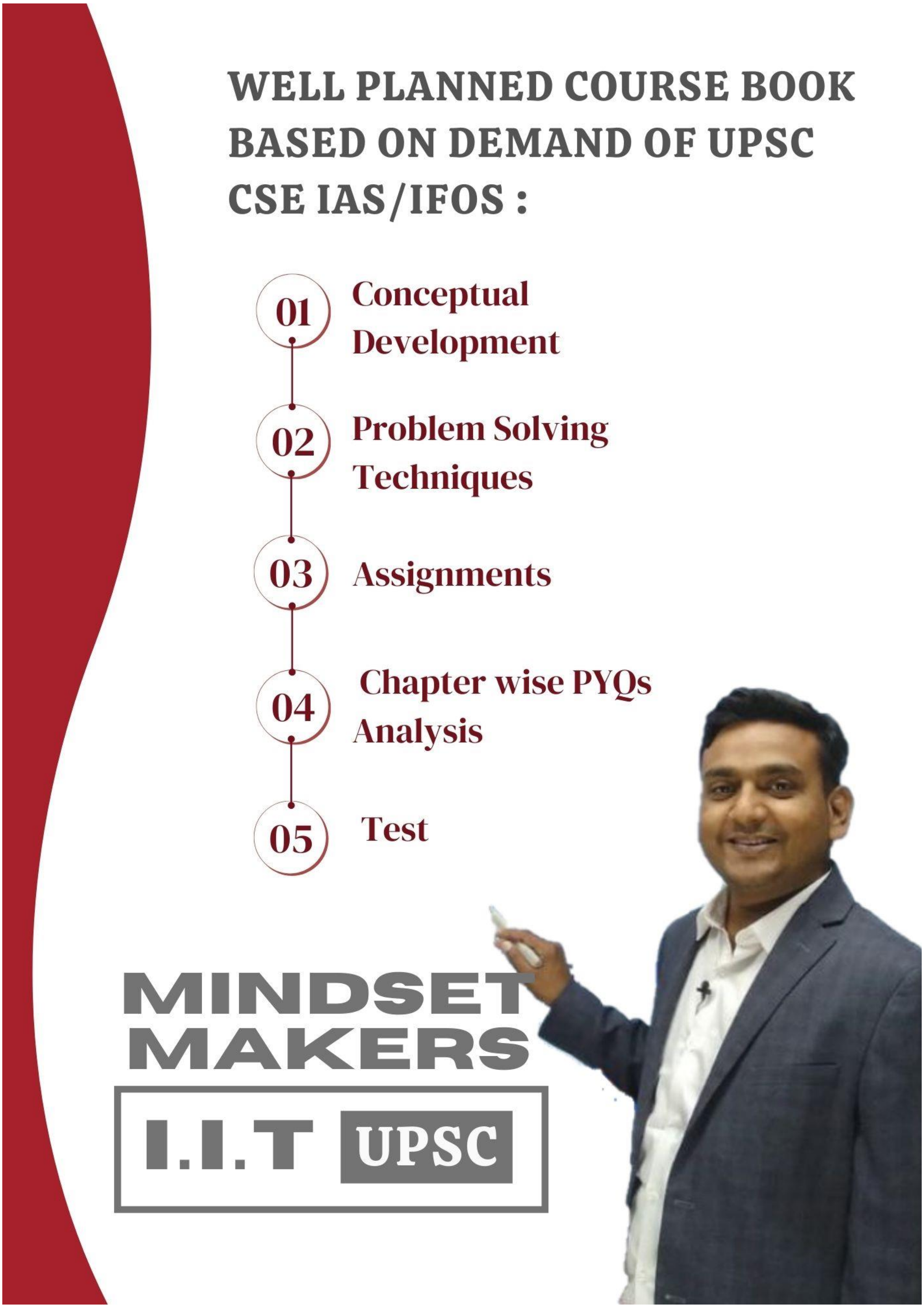
How to Prepare for UPSC (logically with Mathematics Optional)

Mindset Makers for UPSC (Prepare in Right Way)
As an UPSC aspirant with **Science optional (For example Mathematics)**, We need completely a different approach. The interesting part is that if you have clear vision/mindset with such optional then you have high probability to secure the top rank. So is there any sigma rule for such strategy! The answer is – Yes in some sense. You must keep yourself isolated from the thought process where people have negative views about such optionals.

Yes it happens in this UPSC Preparation zone because there is a definite gap of mentors with equal interest in General Studies papers, Essay paper and such optionals. Mentors who are very good at GS and essay , they may not be good science optionals and who are teaching science optional they are experts of their domains mainly. Keeping all such obseravtions in mind , Mindset Makers tries to make easy this preparation with such optional combinations.

For this to happen, we are working in completely offline coaching module at- **YouTube (Upendra Singh : Mindset Makers for UPSC)**. A lecture Series with well planned course structure is going on . Here we teach Mathematics Optional and also do the mindset making for GS and Essay simultaneously with a scientific approach required for this optional combination. You must attend at least one lecture on youtube platform to feel the difference.
Be Part of Disciplined Learning.

Announcement- Teaching Methodologies at a System Based Learning Platform. Books for each topic in Mathematics Optional (to be launched soon). PDFs are available to download.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02 Problem Solving Techniques
03 Assignments
04 Chapter wise PYQs Analysis
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