Test-1 : MATHEMATICS Optional (UPSC CSE & IFoS)
(TOPIC-ORDINARY DIFFERENTIAL EQUATIONS)

Time allowed : Three Hours **Maximum Marks : 250**

Question Paper Specific Instructions:

1- There are eight questions divided in two sections.
2- Candidate has to attempt five question in all.
3- Question nos. 1 and 5 are compulsory and out of remaining , three are to be attempted choosing at least one question from each section.
4- The number of marks carried by a question / part is indicated against it.
5- Answers must be written in the medium authorized.
6- Assume suitable data, if considered necessary, and indicate the same clearly.
7- Unless and otherwise indicated, symbols and notations carry their usual standard meanings.
8- Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partially. Any page or portionof the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION – A

Q1 (a) Show that e^{2x} and e^{3x} are linearly independent solutions of $y''-5y'+6y=0$. Find the solution $y(x)$ with the property $y(0)=0$ and $y'(0)=1$. [10]

(b) By eliminating constants *a* and *b* obtain the differential equation for which $xy = ae^{x} + be^{-x} + x^{2}$ is a solution. [10]

(c) Solve the differential equation $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$. [10]

(d) Show that the general solution of the differential equation $\frac{dy}{dx} + Py = Q \cdot y^n$, where P and Q are constants or function of x alone (and not of y) and n is a constant except 0 and 1; is given by $y^{1-n} e^{(1-n)\int P dx} = \int Q \cdot e^{(1-n)\int P dx} + C$ where C is an arbitrary constant. [10]

(e) Find the family of curves whose tangent form an angle $\pi/4$ with the hyperbola $xy = C$.

[10]

Q2. (a) In a certain city the population gets doubled in 2 years and after 3 years the population is 20,000. Find the number of people initially being living in the city. [15]

(b) Show that the only curves having constant curvature are circles and straight lines. [15]

(c) Find the curve for which sum of reciprocals of the radius vector and the polar subtangent is constant. [20]

Q3. (a) Show that $y = 3e^{2x} + e^{-2x} - 3x$ is the unique solution of the initial value problem, where $y(0) = 4,\ y'(0) = 1$.

(b) The sum of the order first differential equation and the degree of the second differential equation is 9.

I[st]: $x^2\left(\frac{d^2y}{dx^2}\right)^6 + y^{-2/3}\left\{1+\left(\frac{d^3y}{dx^3}\right)^5\right\}^{1/2} + \frac{d^2}{dx^2}\left\{\left(\frac{d^2y}{dx^2}\right)^{-2/3}\right\} = 0$

II[nd]: $dy/dx - 6x = \{ay + bx(dy/dx)\}^{-3/2}, \quad b \neq 0$

Prove or disprove the above statement. [15]

(c) Which of the following pairs of functions is not a linearly independent solutions of $y''+9y=0$?

I. $\sin 3x, \sin 3x-\cos 3x$ 5+5+5+5

II. $\sin 3x+\cos 3x, 3\sin x-4\sin^3 x$

III. $\sin 3x+\cos 3x, 4\cos^3 x-3\cos x$

IV. $\sin 3x, \sin 3x\cos 3x$

Q4. (a) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [10]

(b) Solve $x\cos(y/x)(ydx+xdy)=y\sin(y/x)(xdy-ydx)$ [10]

(c) Solve $x(dy/dx) = y\{\log y - \log x + 1\}$ [10]

(d) Test whether the equation $(x+y)^2 dx-(y^2-2xy-x^2)dy=0$ is exact and hence solve it. [20]

SECTION -B

Q5. (a) Solve $\left(xy^2 + e^{-1/x^3}\right)dx - x^2 y dy = 0$ [10]

(b) Solve the following differential equation $x^2 p^2 - 2xyp + 2y^2 - x^2 = 0$ [10]

(c) Solve $y^2 \log y = x\,py + p^2$ [10]

(d) Solve $\left(x^2+y^2\right)\left(1+p\right)^2-2\left(x+1\right)\left(1+p\right)\left(x+yp\right)+\left(x+yp\right)^2=0$ [10]

(e) Find the general and singular solutions of $3xy = 2px^2 - 2p^2$ [10]

Q6. (a) Solve $x^2 y'' + xy' - y = 0$, given that $x + 1/x$ is one integral by using the method of reduction of order.

(b) Given that the equation $x(1-x)y''+\left(\frac{3}{2}-2x\right)y'-\frac{y}{4}=0$ has a particular integral of the form x^n. Prove that $n=-1/2$ and that the primitive (solution) of the equation is $y=x^{\frac{1}{2}}\left(A+B\sin^{-1}x^{1/2}\right)$ where A and B arbitrary constants. [25]

Q7. (a) Solve $\left(x+2\right)y''-\left(2x+5\right)y'+2y=\left(x+1\right)e^{x}$ [25]

(b) Solve $y''-2\tan x\cdot y'+5y=\sec x\cdot e^{x}$ [25]

Q8. (a) Solve $x^2y''-\left(x^2+2x\right)y'+\left(x+2\right)y=x^3e^x$ [25]

(b) Transform the differential equation $\cos x \cdot y'' + \sin x \cdot y' - 2y\cos^3 x = 2\cos^5 x$ into the one having z as independent variable, where $z = \sin x$ and solve it. [25]

Rough Space