Content: Quantitative Aptitude

TIME AND WORK

Work to be done are usually considered as one unit. It may be constructing a wall, bridge, filling a tank or emptying a tank etc.

There are some basic assumptions that are made in the problems on Time and Work:

1. If a man does a work in certain number of days then it is assumed that he does the work uniformly, that is, he does the same amount of work every day.
2. If there are more than one person involved in a work, then it is assumed that each person does the same amount of work each day.

Unitary Method

The time taken per unit of work or number of persons required to complete a unit work or work completed by unit person in unit time , etc., is what is first calculated while doing the problems on Time and Work and the method adopted in the process is what is called Unitary Method.

Man-days

The concept of man-days is very important and widely used while solving problems on time and work. The number of men multiplied by the number of days that they take to complete the work will give the man-days required to finish a specific work. The total number of man-days for a specific task will remain constant.

EXAMPLES

Example 1. If 18 men take 30 days to complete a work, in how many days can 27 men complete the work?

Solution:

If 18 men can complete the work in 30 days , then the number of man-days required for the work is $18\times30=540$.

If this work is to be done by 27 men then the number of man-days will remain constant, that is, 540.

Hence, the number of days they will take is $540/27=20$ days.

Example 2. 15 men take 10 days to complete a task working at 12 hours a day, how many hours a day should 10 men work to complete the job in 20 days?

Solution. Total men-hours required for the task to be completed= $15\times10\times12=1800$.

When 10 men are required to complete the task in 20 days, the number of men-hours will remain constant.

So the number of hours they should work per day is

$$\frac{15\times10\times12}{10\times20}=9\ hours.$$

Example 3. A and B together can do a piece of work in 12 days and A alone can complete the work in 15 days. How long will B alone take to complete the work?

Solution.

A and B together can do $1/12^{th}$ of work in a day and A alone can complete the $1/15^{th}$ of work in day.

$\therefore$ In one day, work done by B alone is = $\frac{1}{12}-\frac{1}{15}=\frac{1}{60}$

$\therefore$ B alone can complete the work in 60 days.

Example 4. A can do a piece of work in 7 days working 9 hours per day and B can do the same work in 6 days working 7 hours per day. How long will they take to do it, working together 8 hours a day?

Solution.

A can complete the work in 63 hours.

B can complete the work in 42 hours.

A's 1 hour work = 1/63 and B's 1 hour work = 1/42

(A+B)'s 1 hour work = $\frac{1}{63}+\frac{1}{42}=\frac{5}{126}$

Both will finish the work in 126/5 hours.

Hence, number of days= $\frac{126}{5\times 8}=\frac{126}{40}$ days.

Example 5. Anil and Atul can do a task working together in 12 days. Atul alone can do the work in 16 days. Both of them worked together for 3 days and then Atul left. How long will Anil take to complete the remaining task?

Solution.

Work done by both of them in 1 day = 1/12

Work done by both of them in 3 days = $3\times\frac{1}{12}=\frac{1}{4}$

Remaining work = ¾

Work done by Anil in 1 day = $\frac{1}{12}-\frac{1}{16}=\frac{1}{48}$

Since only ¾ of the task is left, Anil will do the remaining work in $\frac{3}{4}\times 48=36$ days.

Example 6. A and B together can do a piece of work in 12 days , B and C can do it in 15 days and C and A can do the same work in 20 days. How long would each take to complete the work?

Solution.

Work done by A& B in 1 day= 1/12

Work done by B& C in 1 day= 1/15

Work done by C& A in 1 day= 1/20

Adding all these we get work done by 2(A+B+C) in 1 day = $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}=\frac{1}{5}$

∴ A, B and C can together finish in 1 day $1/10^{th}$ of the work.

Work done by A in 1 day= work done by A, B, and C in 1 day – work done by B and C in 1 day

= 1/10 – 1/15 = 1/30

∴ A alone can do it in 30 days.

Work done by B in 1 day= work done by A, B, and C in 1 day – work done by Cand A in 1 day

= 1/10 – 1/20= 1/20

∴ B alone can do it in 20 days.

Work done by C in 1 day= work done by A, B, and C in 1 day – work done by A and B in 1 day

= 1/10 – 1/12 = 1/60

∴ C alone can do it in 60 days.

Example 7. A can do a piece of work in 12 days and B is 60% more efficient than A. How many days does B alone take to do the same job?

Solution. Ratio of times taken by A and B = $\left(1+\frac{60}{100}\right):1=1.60:1=8:5$

Suppose B alone takes x days to do the same job.

Then , 8 : 5 : : 12: x

∴ x= $\frac{12\times5}{8}$ = $7\frac{1}{2}$ days.

EXERCISE

1. A is twice as efficient as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

a. 24 days b. 27 days c. 30 days d. 21 days

2. A and B undertake to do a job for Rs. 800. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of C.

a. 400 b. 300 c. 100 d. none

3. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately , A beginning, in how many days the work will be completed? Ans. 10 ¼ days

a. 10 ¼ days b. 10 days c. 12 days d. none

4. 45 men can complete a construction work in 16 days. Six days after they started working , 30 more men joined them. How many days will they now take to complete the remaining work?

a. 6 days b. 8 days c. 10 days d. 12 days

5. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work? Ans

a. 12 ½ days b. 12 days c. 15 days d. 10 days

Ans. 1.(b) 2. (c) 3.(a) 4.(a) 5. (a)

Pipes and Cistern

Facts and formulae

I. **Inlet:** A pipe connected with a tank or a cistern or a reservoir , that fills it is known as an inlet.

II. **Outlet :** A pipe connected with a tank or a cistern or a reservoir , that empties it is known as an outlet.

III. a. If a pipe can fill a full tank in x hours, then part filled in 1 hour = 1/x

b. If a pipe can empty a full tank in y hours, then part emptied in 1 hour = 1/y

c. If a pipe can fill a full tank in x hours, and another pipe can empty the tank in y hours(where y> x), then on opening both the pipes, the net part filled in 1 hour=

$(1/x) - (1/y)$

EXAMPLES

Example 1. Two pipes A and B can fill a tank in 36 and 45 hours respectively. If both the pipes are opened simultaneously how much time will be taken to fill the tank?

Solution.

Part filled by A in 1 hour = 1/36

Part filled by B in 1 hour = 1/45

Part filled by (A+B)in 1 hour= 1/36 + 1/45 = 1/20

Hence, both the pipes together will fill the tank in 20 hours.

Example 2. Two pipes a and B can fill a tank in 10 and 12 hours respectively while a third pipe can empties the full tank in 20 hours. If all the three pipes operate simultaneously , in how much time will the tank be filled?

Solution.

Net part filled in 1 hour= 1/10 + 1/12 - 1/20 = 2/15

$\therefore$ the tank will be full in 15/2 = 7 ½ hours.

Example 3. Two pipes A and B can fill a tank in 36 and 45 minutes respectively. A water pipe C can empty the tank in 30 min. First A and B are opened after 7 minutes C is also opened. In how much time the tank is full?

Solution.

Part filled in 7 minutes = $x\times\left(\frac{1}{24}+\frac{1}{32}\right)+(18-x)\times\frac{1}{24}=1$

Remaining part = 1- 7/20= 13/20.

Net part filled in 1 min when A,B and c are opened= $\frac{1}{36}+\frac{1}{45}-\frac{1}{30}=\frac{1}{60}$

Now, 1/60th part is filled in one minute.

$\therefore$ 13/20th part will be filled in $60\times\frac{13}{20}=39$ minutes.

TIME AND DISTANCE

The basic concepts of Time and Distance are used to solve questions based on

a. Motion in a straight line

b. Relative motion

c. Circular motion

d. Trains

e. Boats

f. Clocks and

g. Races

Before we go into the details, let us first look at some basic concepts related to speed, time and distance.

Speed

Distance covered per unit of time is called speed. Relation between three variables distance, speed and time is :

Distance = Speed $\times$ Time

Or Time = $\dfrac{\text{Distance}}{\text{Speed}}$

✓ If two bodies travel with same speed,

Distance covered $\propto$ Time

✓ If two bodies travel for the same period of time,

Distance covered $\propto$ Speed

✓ If two bodies travel the same distance,

Time $\propto \dfrac{1}{\text{Speed}}$

Conversion of units

- $x\,km/h = x \times \frac{5}{18}\ m/s$
- $x\ m/s = x \times \frac{18}{5} km/h$

Average Speed

Average speed of an object travelling at different speeds is

$$Average\ Speed = \frac{Total\ \text{distance}\ travelled}{Total\ time\ taken}$$

Average speed of an object travelling from point A to B with a speed of u and from B to A with speed of v, then average speed of the object is

$$= \frac{2uv}{u+v}$$

Note that this does not depend upon distance between A and B.

Relative Speed

The speed of one moving object in relation to another moving object is called relative speed of the two objects, that is, it is the speed of one moving object observed from the second moving body.

- ✓ If two objects are moving in the same direction, the relative speed is equal to the difference of the speeds of the two objects.
- ✓ If two objects are moving in the opposite direction, the relative speed is equal to the sum of the speeds of the two objects.

Example 1. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Find the ratio of their speeds.

Solution.

Let the distance covered in 1 leap of the dog be x and that of hare be y. Then,

$$3x = 4y$$
$$x = 4y/3$$
$$4x = \frac{16}{3}y$$

Ratio of speeds of dogs and hare= $4x:5y = \frac{16}{3}y:5y = 16:15$

Example 2. While covering a distance of 36 km, Ram noticed that after walking for 1 hour and 40 minutes, the distance covered by him was 5/7 of the remaining distance. Find his speed.

Solution.

Let the speed of Ram be $x\, km/h$.

Then, the distance covered in 1 hr 40 min = $\frac{5x}{3} km$

Remaining distance= $\left(36 - \frac{5x}{3}\right) km$

Given,

$$\frac{5x}{3} = \frac{5}{7}\left(36 - \frac{5x}{3}\right)$$
$$\therefore x = 9\, km/h$$

Example 4. Walking at 5/6 of usual speed, Shyam is 10 minutes late to reach his office. Find the usual time taken by him to reach his office.

Solution.

Let usual time= t

So as the new speed is 5/6 of the usual speed, so the

new time taken= $\frac{6t}{5}$

Given,

$\frac{6t}{5}$ - t = 10 minutes

$\therefore\ usual\ time\ \ t = 50\,\text{min}$

Example 5. If Mohan walks at the rate of 5km/h he misses a train by 7 minutes. However, If he walks at the rate of 6km/h, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Solution.

Let the distance be x km.

Difference in the time at two speeds = 12 min = 1/5 hr

Hence,

$$\frac{x}{5} - \frac{x}{6} = \frac{1}{5}$$

$$\frac{6x - 5x}{30} = \frac{1}{5}$$

$$\therefore x = 6\,km$$

Trains

- ✓ Time taken by a train of length l meters to pass a pole or a standing man or a signal post is equal to to the time taken by the train to cover l meters.
- ✓ Time taken by a train of length l meters to pass a stationary object of length b meters is equal to to the time taken by the train to cover $(l + b)$ meters.

<u>EXAMPLES</u>

Example 1. A train 100m long is running at the speed of 30km/h. Find the time taken by it to pass a man standing near the railway line?

Solution.

Speed of the train = $30 \times \frac{5}{18} = \frac{25}{3} m/s$

Distance moved in passing the standing man= 100m

Hence, required time = $\dfrac{100}{\frac{25}{3}} = 12\,\text{sec}$

Example 2. A train is moving at a speed of 132 km/h. If the length of the train is 110 meters, how long will it take to cross a railway platform 165 meters long?

Solution.

Speed of the train = $132 \times \frac{5}{18} = \frac{110}{3} m/s$

Distance covered in passing the platform= 110+ 165= 275 m

Time taken = $\dfrac{275}{\frac{110}{3}}$ = 7 ½ sec

Example 3. A man standing on a bridge which is 180m long finds that a train crosses the bridge in 20 seconds but crosses him in 8 seconds. Find the length and the speed of the train?

Solution.

Let the length of the train be x meters.

Then,

The train crosses x meters in 8 seconds and (x +180) meters in 20 seconds.

$$\therefore \quad \frac{x}{8} = \frac{x+180}{20}$$

Or, x = 120 m.

Speed of the train = 120/8 m/s = 54 km/h

Example 4. Two trains 140 meters and 160 meters in length are running towards each other on parallel lines, one at the rate of 42km/h and another at 48 km/h. In what time will they cross each other from the moment they meet?

Solution.

Relative speed of the train = 42+48= 90 km/h = 25m/s

Time taken by the trains to pass each other= time to cover 140+ 160= 300 meters

= 300/ 25= 12 seconds.

Boats and Streams

a. Moving with the stream(current)

When a boat is moving in the same direction as the stream or water current, the boat is said to be moving with the stream/current.

Speed of the boat with the stream= Speed of the boat in still water + Speed of the stream

b. Moving against the stream(current)

When a boat is moving in the opposite direction as the stream or water current, the boat is said to be moving against the stream/current**.**

Speed of the boat against the stream**=** Speed of the boat in still water - Speed of the stream

These two speeds are relative speeds.

Example 1. A man can row upstream at 6km/h and downstream at 10km/h. Find man's speed in still water and the speed of the current?

Solution.

Speed in still water = ½(10+6)= 8km/hr

Speed of the current = ½(10-6)= 2 km/hr

Example 2. A man can row 8 km in one hour in still water. If the speed of the water current is 2km/hr and it takes 3 hours for him to go to a new place and return, find the distance from the starting point to the new place.

Solution.

Let the distance be x km.

Upstream speed= 8-2= 6km/hr

Downstream speed= 8+2= 10 km/hr

Total time taken = x/6 + x/10= 3 (given)

Solving we get x= 180/16= 11 ¼ km.

Example 3. A boat can row 2 ½ times the distance down the stream than up the stream in the same time. If the speed of the current is 3km/hr, find the speed of the boat in still water?

Solution.

As the distance covered down the stream is 2 ½ times that covered up the stream, the speed down the stream will also be 2 ½ times the speed up the stream.

Let the speeds of the boat in still water be v,

Then, $\frac{v+3}{v-3}=\frac{5}{2}$

⇨ v = 7 km/h.

Races and Circular Tracks

When two persons P and Q are running a race, they can start the race at the same time or one of them may start a little later.

a. Suppose P starts the race and after 5 seconds Q starts, then we say that P has a "start" of 5 seconds.

b. Again, suppose P starts first and then when P has covered a distance of 10 meters, Q starts. Then, we say that P has a “start” of 10 meters.

When two or more persons are running around a circular track, starting at the same point and starting at the same time, then we are interested in two things:

a. When will they meet for the first time and
b. When will they meet for the first time at the starting point.

To solve the problems related to circular tracks, following points are to be kept in mind:

a. When two persons are running in opposite direction:
✓ The relative speed is the sum of the speeds
✓ From one meeting pint to the next meeting point, the two runners cover a distance equal to the length of the track.
b. When two persons are running in the same direction:
✓ The relative speed is the difference of the speeds
✓ From one meeting pint to the next meeting point, the faster covers one complete round more than the slower one.

EXAMPLES

Example 1. In a 1000m race, P beats Q by 28 meters or 7 seconds. Find P’s time over the course?

Solution.

It is evident that Q covers 28 meters in 7 seconds.

So time taken by Q to cover the course= 250 seconds

And time take by P to cover= 250-7= 243 seconds.

Example 2. P runs 1 ¾ times as fast as Q. If P gives a start of 84m to Q, how far must winning post be so that P and Q might reach it at the same time?

Solution.

Ratio of the speeds of P and Q = 7:4

So in a race of 7m, P gains 3m over Q.

∵ 3m are gains by P in a race of 7m

∴ 84m are gains by P in a race of $\frac{7}{3} \times 84 = 196$ m.

So, winning post must be 196m away from the starting point.

Example 3. In a race on circular track of length 1200 meters, P and Q start with the speeds 18km/h and 27km/h respectively starting at the same point. When will they meet for the first time at the starting point when running

a. In the same direction

b. In the opposite direction

Solution.

Speed of P = $18\times\frac{5}{18}=5m/s$

Speed of Q = $27\times\frac{5}{18}=7.5m/s$

Time take by P to complete a round= 1200/5= 240 s

Time take by P to complete a round = 1200/ 7.5= 160 s

a. They will meet at the starting pint at a time which is the LCM of the timings taken by each of them to complete one full round

LCM{ 160 s and 240 s } = 480 s.

b. They will meet at the starting pint at a time which is the LCM of the timings taken by each of them to complete one full round

LCM{ 160 s and 240 s } = 480 s.

Example 4. A, B, and C with respective speed of 9, 18 and 27km/hr run around a circular track 1200m long. If they started at the same time from the same point and run in the same direction, when will they meet for the first time?

Solution.

Speed of A, $v_A = \frac{v+3}{v-3}=\frac{5}{2}$

Speed of B, $v_B = 18\times\frac{5}{18}=5\,m/s$

Speed of C, $v_C = 27\times\frac{5}{18}=7.5\,m/s$

They will meet for the first time at a time which is LCM of at the starting point

$\left\{\frac{1200}{5-2.5},\frac{1200}{7.5-5}\right\}=480\,\text{seconds}$, i.e., 8 minutes after they start.

PERCENTAGES

In mathematics, percent means 'per cent', that is, 'out of 100'. Hence result of any division in which divisor is 100 is simply a percentage. And the divisor 100 is denoted by the symbol '$\%$'.

For example, $\frac{15}{100}=15\%$

$$\frac{34}{100}=34\%$$

$$\frac{x}{100}=x\%$$

Expressing $\%$ as a fraction

Any percentage can be written as a decimal fraction by dividing the percentage figure by 100.

for example

$$39\%=\frac{39}{100}=0.39$$

$$60\%=60 \text{ out of } 100$$

$$=\frac{60}{100}=\frac{3}{5} \text{ or } 0.60.$$

Utility

The concept of percentage is useful in that respects where we are concerned for comparing data and information such as profits, growth rates, performance, etc.

Example (comparison)

In an examination John scores 20 out of 30 and Kathy scores 40 out of 80. Who has performed better ?

Sol.

By comparing their works we don't get a clear picture of their actual performance for that, we need to express the scores in percent values.

Thus, John gets $\frac{20}{30}\times 100=66.66\%$

and Kathy gets $\frac{40}{80}\times 100=50\%$

Hence, we see that John performance is better than Kathy.

Percentage Increase(decrease)

Increase or decrease of a quantity with respect to original quantity when expressed out of hundred is percentage increase or decrease. That is,

$$\textit{Percentage increase } = \frac{\text{Actual increase}}{\text{Original quantity}}\times 100$$

$$\textit{Percentage decrease} = \frac{\text{Actual decrease}}{\text{Original quantity}} \times 100$$

Example

If the production of wheat when up from $250\,mt$ in 2015 to $275\,mt$ in 2016, then percentage increase in wheat production from 2015 to 2016, is

Actual increase $= (275 - 250)\,mt$

$= 25mt$

Percentage increase $= \dfrac{\text{Increase from 2015 to 2016}}{\text{Actual production of wheet in 2015}}$

$$= \frac{25}{250} \times 100 = 10\%$$

Scenario1:

Whenever there are cases of percentage increase or decrease on a quantity instead of calculating the actual increase (decrease) and then adding(subtraction) from the original, we can directly calculate the new value.

For example,

(a) If the increase on a figure of 450 is 20% the new quantity is

$$1.20 \times 450 = 54$$

Here, $1.20 = 1 + 0.20, 0.20$ being the decimal equivalent of 20%.

(b) If the production in 2020 is given as $600\,MT$ and increase from 2019 to 2020 is given to be 25% then the production in 2019 will be equal to

$$\frac{600}{1.25} = 480\,MT$$

Where $1.25 = 1 + 0.25, 0.25$ being the decimal equivalent of 25%.

Scenario-2: (Successive changes)

If there are successive increases of $p\%$, $q\%$ and $r\%$ in three stages, the effective percentage increase is

$$\left\{\left(1+\frac{p}{100}\right)\left(1+\frac{q}{100}\right)\left(1+\frac{r}{100}\right)-1\right\}\times 100$$

If one or more of p, q and r are decrease percentage figures, then it will be taken as a negative figure and not as positive figure. Also, if the resultant figure turns out to be negative, it means it is a case of decrease.

EXAMPLES

Example 1: 80% of a particular total is 720. What is 92% of the total?

Sol. Let the total be x.

Then, 80% of $x = 720$

Or $\frac{80}{100} \times x = 720$

$\therefore x = 720 \times \frac{100}{80} = 900$

So, 92% of $900 = \frac{92}{100} \times 900 = 828$

Alternative solution

$\because 80\%$ is equal to 720

$\therefore 92\%$ is equal to $\frac{720}{80} \times 92 = 782$

Example 2: If $A's$ salary is 20% less than $B's$ salary then by what percent is $B's$ salary more than that of A?

Sol. Let $B's$ salary be $Rs.100$.

$A's$ salary $= 100 - 20 = Rs.80$

$B's$ salary is 20 more than that of $A's$.

So, the required percentage $= \left(\frac{20}{80} \times 100\right)\%$

$= 25\%$

$\therefore B's$ salary is 25% more than $A's$ salary.

Example 3: The rate of sugar is decreased by 20%. By what percent its consumption must be increased so that the expenditure on it increases by 10%?

Sol. Let the rate of sugar be $Rs.\ x/kg$ and

Consumption $= y\ kg$

So, expenditure $= Rs.xy$

Now, new rate of sugar $= x \times \left(1 - \frac{20}{100}\right)$

$= 0.80x/kg$.

Let the new consumption $= C\ kg$

$$\text{New expenditure} = xy\left(1+\frac{10}{100}\right) \text{ rupees.}$$

$$= \frac{11xy}{10}$$

Thus, $0.80x \times c = 1.10xy$

$\therefore c = \frac{110}{80} y \ \ kg$

Change in consumption $= c - y$

$$= \frac{110}{80} y - y = \frac{30}{80} y$$

% change (increase) $= \dfrac{\frac{30}{80} y}{y} \times 100\,\text{s}$

$= \frac{300}{8} = 37.5\%$

Example 4: The height of a triangle is increased by 20% and base is decreased by 10%. What is the increase / decrease in the area?

Sol. Let the height be h end the base be b.

Initial area $= \frac{1}{2} bh$

New height $= h\left(1+\frac{20}{100}\right) = 1.20\,h$

New base $= b\left(1-\frac{10}{100}\right) = 0.90\,h$

$\therefore$ New area $= \frac{1}{2}(0.90\,b)(1.20h)$

$= \frac{1}{2} bh(1.08)$

Increase in area $= \frac{1}{2} bh(1.08) - \frac{1}{2} bh$

$= (0.08)\frac{bh}{2} = 0.04\,bh$

$\therefore$ % increase in area $= \dfrac{(0.04)bh}{(0.5)bh} \times 100$

$= 8\%$

Example 5: The length, breadth and height of a tank is increased by 10%, 20% and 25% respectively. Find the percentage change in the volume of the tank.

Sol. Let the length, breadth and height be l, b and h unit.

New length $= l\left(1+\frac{10}{100}\right)$

$= \frac{110}{100} l$

New breadth $= \frac{120}{100} b$

New height $= \frac{125}{100} h$

$\therefore$ New volume $= 1.10 l \times 1.20 b \times 1.25 h$

$= 1.65\ lbh$

Old volume $= lbh$

Increase in volume $= 1.65 lbh - lbh$

$= 0.65 lbh$

$\%$ increase in volume $= \frac{0.65 lbh}{lbh} \times 100$

$= 65\%$

Example 6: In an examination, 80% students passed in Maths, 70% in Biology while 15% failed in both subjects. If 390 students passed in both the subjects. Find the total number of students who appeared in the examination.

Sol. Let the total number of students who appeared in the examination be 100.

So, the number of students who passed in Maths $= 80$

$\therefore$ Number of students who failed in maths $= 100 - 80 = 20$

Similarly number of students who failed in Biology $= 100 - 70 = 30$.

Total failed students $=$ no. of students who failed in maths

$+$ no of students who failed in biology

$-$ no of students who failed in both subjects

$= 20 + 30 - 15$

$= 35$

$\therefore$ Total number of students who passed

$= 100 - 35 = 65$

$\because$ 65 students passed in 100 students

$\therefore$ 390 students passed in $\frac{100}{65} \times 390$

$= 600$

$\therefore$ Total number of students who appeared in examination $= 600$.

EXERCISE

1. In an election between two candidates A and B, a secured 60% valid votes more than B, and 9% of the total votes were invalid. Find the percentage of the total votes in favor of A.

a. 15.5% b. 75.5 % c. 75% d. 25%

2. Ram suffered a loss of 15% by selling his table. Had he sold it for ₹ 100 more, he would have made a profit of 5%. Find the cost price of the table.

a. 500 b. 400 c. 425 c. None

3. Ram and Shyam save 20% and 30% of their income. If their expenditure are equal and Shyam's income is ₹ 5600, then find Ram's savings (in ₹)

a. 490 b. 980 c. 1470 d. 1225

4. In a test, by obtaining $41\frac{2}{3}$% of the maximum marks, John obtains 10 marks more than the pass marks. By scoring 30% of the maximum marks, Kathy scores 4 marks less than the pass marks. Find the actual pass percentage.

a. $11\frac{2}{3}\%$ b. $22\frac{1}{6}\%$ c. $33\frac{1}{3}\%$ d. $66\frac{2}{3}\%$

5. In an examination, 80% of the students passed in English, 85% in mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.

a. 400 b. 200 c. 300 d. 500

6. A spends 75% of his income. His income is increased by 20% and his expenditure increases by 10 % . Find the percentage increase in his savings.

a. 25% b. 50% c. 20 % d. 12.5 %

ANSWERS

Ans. 1.(b) 2. (a) 3. (b) 4.(c) 5.(a) 6.()

PROFIT & LOSS

Profit and Loss are part of business transactions. Transactions involves sales and purchases. And each transaction involves – a seller and a buyer. Certain terms which are part and parcel of every transaction are –

(i) Cost price (CP).

This is a price at which an article/object is purchased.

(ii) Selling price (SP).

The price at which an article/object is sold is the seller price of that article/object.

(iii) Profit /gain :

– It is earned by seller.

– It is a case when a seller sells his article at a price more than its cost price. That is,

When Selling price $(SP) > $ Cost price (CP)

$\therefore$ The profit is gain over cost price of an article.

Profit $=$ Seller price $(SP) -$ Cost price (CP)

(IV) Loss

Likewise, the loss is a gain over cost price but in such situations cost price is more than seller price of an object. Thus,

Loss $=$ Cost price $(CP) -$ Sellings price (SP)

Percentage Profit

Profit earned over cost price when expressed in "out of 100" is percentage profit in a particular transaction. That is,

$$\%\,\text{Profit} = \frac{SP - CP}{CP} \times 100$$

Note: Percentage profit is always calculated on cost price.

Some mathematical formulations :

Case 1:

Knowns : Cost Price (CP) & Profit %

Unknowns: Selling Price (SP).

$$\therefore SP = CP\left(1 + \frac{\text{profit}\,\%}{100}\right)$$

Case2:

Knowns : Seller Price (SP) & Profit %

Unknowns: Cost Price (CP).

$$\therefore\ CP = \frac{SP}{\left(1+\frac{\text{Profit\%}}{100}\right)}$$

Percentage Loss

Similar to the case of Percentage Profit, here also the Loss expressed in terms of "out of 100" is Percentage Loss. That is,

$$\%\,\text{Loss} = \frac{CP - SP}{CP}\times 100$$

Case1:

Knowns : Cost Price (CP) & Loss % .

Unknowns : Selling Price (SP).

$$SP = CP\left(1-\frac{\text{Loss\%}}{100}\right)$$

Case 2:

Knowns : Seller price (SP) and Loss % .

Unknowns : Cost price .

$$\therefore CP = \frac{SP}{\left(1-\frac{\text{Loss\%}}{100}\right)}$$

Note: Percentage Loss is also always calculated on cost price.

Overheads

All the overheads incurred on transportations, repairs, maintenance, etc., (if any) are simply overheads. These overheads are part of the cast price of the article in transaction.

Mark up and Marked price

(a) While selling an article a seller sometimes add a certain percentage on the cost price. This is known as percentage mark-up and the price so obtained after addition are called marked price. Thus,

$$\text{Cost price} + \text{mark-up} = \text{Marked price}$$

or, CP + % mark-up on $CP =$ Marked price.

If an article is sold at its marked price, then seller price $=$ marked price.

(b) ***Discount*:**

Sometimes a trader/ seller gives discount, such discount he does on marked price and the price at which it is sold after discount is known is 'discounted price'. Thus,

$$\text{Seller price} = \text{Marked price} - \% \text{ Discount.}$$

EXAMPLES

Example 1. Mohan purchased a TV-Set for $Rs.4000$. He incurrent $Rs.500$ for its transportation. Finally, he sold the TV-Set at $Rs.6000$. Find his profit percentage.

Sol.

$$\text{Cost prices } (CP) \text{ of TV-Set } = Rs.4000 + Rs.500$$

$$= Rs.\ 4500$$

(Overheads in the form of transportation will be part of cost price).

Seller price (SP) of the TV-Set $= Rs.6000$.

$\because CP < SP$, so a case of profit

The amount of profit $= SP - CP$

$$= Rs.6000 - Rs.\ 4500$$

$$= Rs.1500$$

$$\therefore \text{ Profit percentage } = \frac{\text{Profit}}{CP} \times 100\%$$

$$= \frac{Rs.1500}{Rs.4500} \times 100$$

$$= \frac{100}{3}\%$$

$$= 33\frac{1}{3}\%$$

$\therefore$ Mohan earned a profit of $33\frac{1}{3}\%$.

Example 2. If by selling 25 pens, Ajay makes a profit equal to the cost price of 5 pens. Find his profit percentage.

Sol. Let us assume the cost price of each pen $Rs.\ 1$.

$\therefore CP$ of 25 pen $= Rs.25$

Profit $= CP$ of 5 pens $= 5 \times Rs.1 = Rs.5$

$$\therefore \text{ Profit percentage } = \left(\frac{\text{Profit}}{CP} \times 100\right)\%$$

$$= \left(\frac{Rs.5}{Rs.25} \times 100 \right) \%$$

$$= 20\%$$

$\therefore$ Ajay's profit is 20%.

Example 3. Kathy sold two books for $R\,480$ each, there by having gain of 20% on one book and loss 20% on the other book. Find her overall loss or gain percent.

Sol. Selling price of the first book $= Rs.480$

Profit $= 20\%$

$$\therefore CP = \frac{SP}{\left(1+\frac{\text{Profit}\,\%}{100}\right)} = \frac{Rs.480}{\left(1+\frac{20}{100}\right)} = Rs.s\,400$$

Selling price of the second book $= Rs.480$

Loss $= 20\%$

$$\therefore CP = \frac{SP}{\left(1-\frac{\text{Loss}\,\%}{100}\right)} = \frac{Rs.480}{\left(1-\frac{20}{100}\right)} = Rs.600$$

So, The total selling price of books $= Rs.480 + Rs.480$

$= Rs.960$

The total cost price of the books

$= Rs.400 + Rs.600$

$= Rs.1000$.

Since, $SP < CP$ This is a case of loss.

$\therefore$ Loss $= Rs.1000 - Rs.960$

$= R\,40$

$$\text{Loss percentage} = \left(\frac{\text{Loss}}{CP} \times 100 \right) \%$$

$$= \left(\frac{Rs.40}{Rs.1000} \times 100 \right) \%$$

$$= 4\%$$

Hence, Kathy's Loss $= 4\%$.

Example 4. By selling apple at the rate $Rs.52$ per kg, a shopkeeper gains 30%. At what rate he should sell it to gain 40%?

Sol.

The selling price $= Rs.52/kg$.

Gain $= 30\%$

$$\therefore CP = \frac{Rs.52}{\left(1+\frac{30}{100}\right)} = \frac{Rs.52}{\frac{130}{100}} = Rs.40$$

Now, $CP = Rs.40$ and gain is 40%.

$$\therefore SP = CP\left(1+\frac{\text{Profit}\,\%}{100}\right) = Rs.40\times\left(1+\frac{40}{100}\right)$$

$$= Rs.56$$

$\therefore$ To have gain of 40%, shopkeeper has to sell it at $Rs.56$ per kg.

Example 5. A shopkeeper marked the price of books at 50% above the cost price and then gives discount of 50% before seller them. If selling price of a book *was* $Rs.375$. What is the profit percentage he made or loss percentage incurred in this transaction. Choose the correct option:

(a) $33\frac{1}{3}\%$ Profit (b) 25% Profit (c) 25% Loss (d) No Loss or Profit

Sol. Let the cost price $= Rs.C$

$$\therefore \text{ Marked price } (m) = C\left(1+\frac{50}{100}\right) = s\,\frac{150C}{100}$$

$$= 1.50C$$

$$\text{Discount } = \frac{50}{100}(1.50C) = 0.75C$$

$$\therefore \text{ Selling price } (SP) = 1.50C - 0.75C$$

$$= 0.75C$$

Given,

$0.75C = 375$

$$\therefore C = \frac{375}{0.75} = 500$$

Selling price $= 375$

Cost price $= 500$

$\therefore$ A loss was incurred by him.

The Loss $= 500 - 375$

$= Rs.\,125$

$\therefore$ Loss % $= \frac{125}{500} \times 100 = 25\%$

Example 6. A reduction of 20% in the price of sugar enables Rakesh to buy $14\,kg$ more for $Rs.280$. Find the reduced price per kilogram.

(a) $Rs.4$ (b) $Rs.5$ (c) $Rs.3.2$ (d) None of these

Sol. Let the price (before reduction) be $Rs.\,x/kg$.

Sugar purchased at this rate $= \frac{280}{x} kg$.

$$\text{Reduced price } = x \times \left(1 - \frac{20}{100}\right) = \frac{80x}{100} kg.$$

Now, sugar purchased for $Rs.\ 280$,

$$= \frac{280}{\frac{80x}{100}}$$

Given,

$$\frac{280}{\frac{80x}{100}} = \frac{280}{x} + 14$$

or, $x = Rs.\ 5/kg$.

$\therefore$ Reduced price per kilogram $= Rs.5 \times \left(1 - \frac{20}{100}\right) = Rs.4/kg$

Example 7. A dishonest shopkeeper claims that he sells at the cost price but uses a $900\,gm$ weight instead of a 1 kilogram weight. Find the percent of profit by hine.

(a) 12.5% (b) 10% (c) 11.11% (d) None of these

Sol. Let the cost price of an article of $1000\,gm$ is $Rs.\,1000$.

So, the cost price of $900\,gm = Rs.\ 900$.

But selling price of $900\,gm = Rs.\ 1000$.

Profit $= Rs.1000 - Rs.900$

$= Rs.100$

Profit percentage $= \frac{100}{900} \times 100 = 11.11\%$

EXERCISE

1. By selling 33 articles, Ram gains the selling price of 11 articles. Find the gain percent.

a. 25 % b. 50 % c. 33.33% d. None

2. A grocer purchased 80 kg of rice at ₹ 13.50 per kg and mixed it with 120 kg of rice with ₹ 16 per kg. At what rate should he sell the mixture to have gain of 16%.

a. ₹18 per kg b. ₹ 17.40 per kg c. ₹ 18 per kg d. None

3. A dealer professes to sell his good at the cost price but uses a weight 960 gm for a kg weight. Find his gain percent.

a. $4\frac{1}{6}\%$ b. 5 % c. 9 % d. None

4. A man bought a cycle and a bike for ₹ 3000. He sold the cycle at a gain of 20% and the bike at a loss of 10%, thereby gains 2% on the whole transaction. Find the cost of the cycle.

a. ₹1200 b. ₹1500 c. ₹1600 d . ₹1350

5. Ramu sold his two books in such a manner that selling price of first book is equal to the cost price of the second book . He sold the first book at 10% profit and the second at loss of 10%. Find his overall gain/ loss percentage.

a. $\frac{5}{21}\%$ profit b. $\frac{5}{21}\%$ loss c. $\frac{10}{21}\%$ profit d. $\frac{10}{21}\%$ loss

6. In a certain season a football club had won 30% of the first 60 matches it played. Find the minimum possible number of additional matches it should play to have success rate of 44% in that season.

a. 20 b. 25 c. 15 d. 30

ANSWERS

Ans. 1.(b) 2. (b) 3. (a) 4.(a) 5.(d) 6. (c)

Averages – Mixtures

The average of a set of numbers is a measure of the central tendency of the whole numbers in the set. That is, it is an estimate of where the centre point of a set of numbers lies.

Average

That is, average of n numbers $x_1, x_2, x_3, \ldots, x_n$ is

$$A_n = (x_1 + x_2 + x_3 + \ldots + x_n)/n$$

This means, also, $A_n \times n$ = total of the set of numbers.

Weighted average

When we have two or more grouped data whose individual averages are known, then combined average of all the elements is calculated as follows:

Suppose we have k groups with averages $A_1, A_2, \ldots, A_k$ and have $n_1, n_2, \ldots, n_n$ elements then the weighted average is given by the formula:

$$A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots + n_nA_n}{n_1 + n_2 + n_3 + \ldots + n_n}$$

Points to remember

I. If the value of each item is increased/decreased by the same value C, then average of the items also increases/ decreases by C.

II. If the value of each item is multiplied/ divided by the same value C, then average of the items also gets multiplied/ divided by C.

III. The average of a group of items will always lie between the smallest value in the group and the largest value in the group.

EXAMPLES

Example 1. Ram purchased three kg of wheat at ₹ 7 per kg, two kg of wheat at ₹ 8.50, and one kg at ₹ 10 per kg. what is the average cost of the wheat per kg purchased by him?

Sol.

Cost of 3 kg wheat= 3× 7 = 21

Cost of 2 kg wheat= 2× 8.5 = 17

Cost of 1 kg wheat= 1× 10 = 10

Total cost of 6 kg of wheat = ₹ 48.

Average cost = 48/6= ₹ 8 per kg.

Example 2. The average scores of three sections of a class were 76, 79, and 80 respectively. If the number of students in these three sections were 70, 60, and 55 respectively, find the average marks of the class.

Sol.

Using the weighted average rule, the overall average is

$$\frac{76\times70+79\times60+80\times55}{70+60+55}=78.16$$

Example 3. The average age of a class of students is 34 years. If five new students with an average age of 30 years join the group, the average of the entire class becomes 32 years. How many people were there in the class initially?

Sol.

Suppose there were n students in the class initially.

Then, the total age of the class after 5 new students join the class = 34 n + (5×30)

But this is also equal to $32(n+5)$

$\therefore$ $32(n+5) = 34n + (5\times30)$

So, n = 5 , so there were 5 students initially in the class.

Example 4. If 15 litres of 20% alcohol is mixed with 12 litres of 24% alcohol, then find the concentration of the resulting solution.

Sol.

Applying the weighted average rule,

Concentration of the resulting solution= $\frac{20\times15+12\times24}{15+12}=\frac{588}{27}=21\frac{7}{9}\%$

ALLIGATION OR MIXTURE

I. Alligation

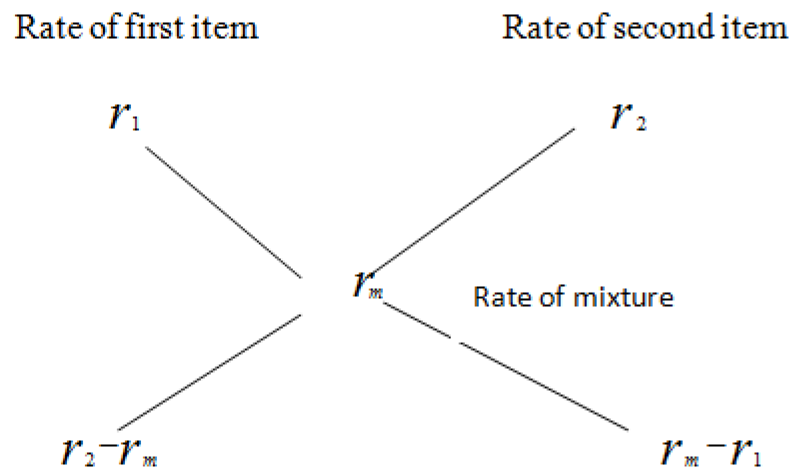
It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

II. Mean Price

The cost of a net quantity of the mixture is called the mean price.

III. Rule of Alligation

If two ingredients are mixed then



Quantity of first item : quantity of second item = $(r_2 - r_m) : (r_m - r_1)$

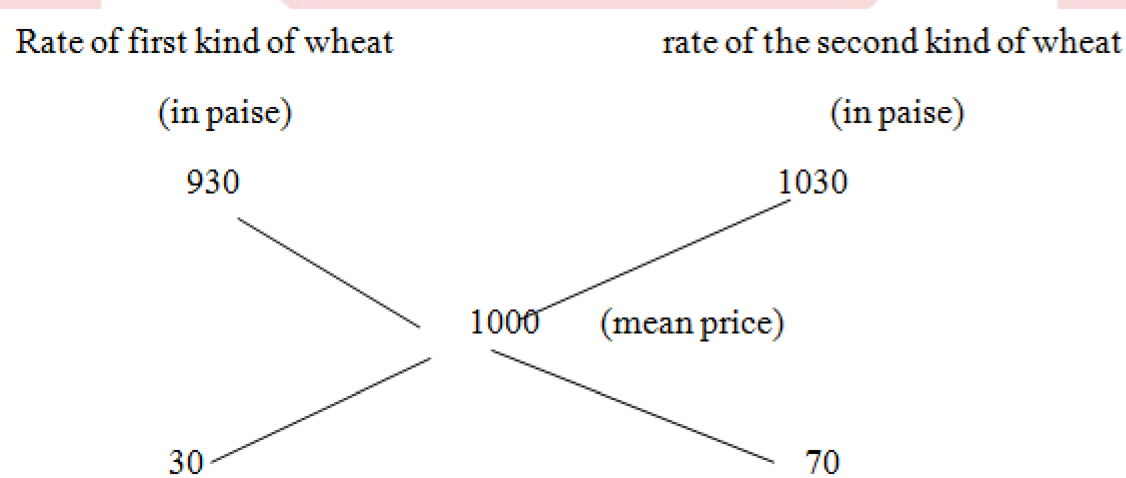
IV. Suppose a container contains x units of pure liquid from which y units are taken out and replaced by water. After n such operations the quantity of pure liquid is

$$x\left(1-\frac{y}{x}\right)^n \text{ units}$$

EXAMPLES

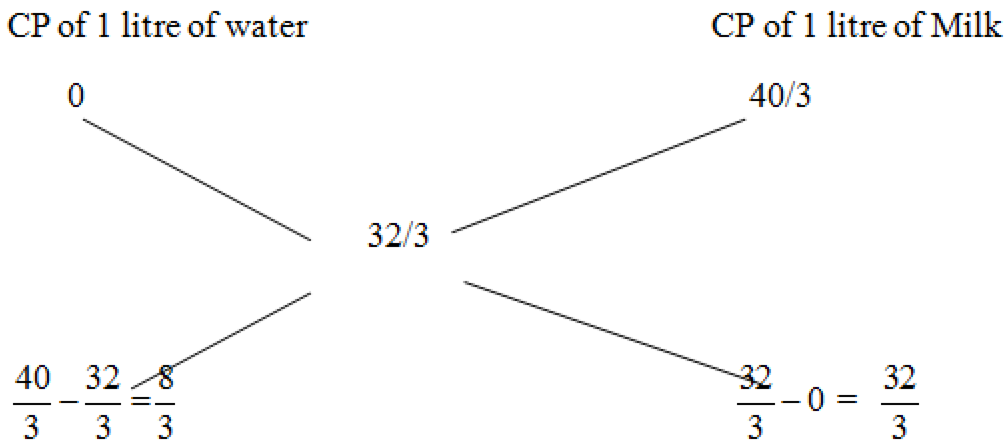
Example 1. In what ratio must wheat at ₹ 9.30 per kg be mixed with wheat at the rate ₹ 10:30 per kg so that the mixture be worth ₹10 per kg?

Sol. Using the rule of alligation we have,



So, the required ratio= 30:70 = 3:7.

Example 2. How much water must be added to 60 liters of milk of the rate ₹ $13\frac{1}{3}$ per litre so as to have mixture worth ₹$10\frac{2}{3}$ a litre?



$\therefore$ *ratio of water and milk* $= \frac{8}{3} : \frac{32}{3} = 1:4$

So, the quantity of water to be added to 60 litres of milk = $\frac{1}{4} \times 60 = 15$ litre.

EXERCISE

1. The average weight of 5 persons is decreased by 3 kg when one of them weighing 150 kg is relpaced by another person. This new person is again replaced by another person whose weight is 30 kg lower than the person to be replaced. What is the total change in the average due to this dual change?

(a) 6 kg (b) 9 kg (c) 12 kg (d) 15 kg

2. The average weight of A, B, C and D is $50\ kg$. If the average age of A and C is 30, the average age of B and D is

(a) $120\,kg$ (b) $70kg$ (c) $90kg$ (d) $180\,kg$

3. The average salary of 11 person is 50. If the average salary of first six persons is 49 and that of last six is 52, find the salary of the sixth person

(a) 30 (b) 40 (c) 50 (d) 56

4. In a class there are 20 students whose average age is decreased by 2 months when one student aged 18 years is replaced by a new student. The age of the new student is

(a) 5 *years* 2*months* (b) 6 *years* 3*months*

(c) 7 *years* 8months (d) 8 *years* 8*months*

5. In an examination, an students average marks was 63 per paper. If he had obtained 20 more marks in Geography paper and 2 more marks for history paper his average would have been 65. How many papers were there in the examination?

(a) 7 (b) 11 (c)13 (d)15

6. A total amount of ₹ 194 was collected by contributions from boys and girls of strength 150. If each boy contributions was ₹1.2 and each girl ₹1.4, then find the number of boys and girls who contributed.

7. In a mixture of alcohol and water of 40 liters, alcohol is 72%. How much water should be added to this mixture to make it a mixture in which alcohol forms 60%?

a. 10 liters b. 8 liters c. 12 liters d. 6 liters

8. 6 kg of wheat costing ₹ 15 per kg is added to 9 kg of wheat costing ₹12 per kg. at what price should the mixture be sold so that there is no profit or loss? (in ₹ per kg)

a. 12.50 b. 13 c. 13.20 d. 14

9. The average of a sequence of 6 consecutive integers is n. find the average of the next 8 consecutive integers.

a. n+10 b. n+7 c. n+14 d. n+2

10. A milkman professes to sell milk at cost price but he makes $33\frac{1}{3}\%$ profit by mixing it with water. What is the water content in the milk solution?

a. 20% b. $33\frac{1}{3}\%$ c. 25% d. 30%

<u>ANSWER</u>

Ans. 1. (b) 2.(b) 3.(d) 4.(d) 5. (b) 6. Ans. 70, 80 7. (b) 8.(c) 9. (b) 10. (c)

Ratio, Proportion and Variation

The concept of ratio, proportion and variation is very important in the area of data interpretation, where the ratio change and ratio comparisons are very important part of questions.

Ratio

When comparing any two numbers, sometimes, it is necessary to find out what multiple, part or parts one quantity is of the other. Only quantities of the same kind can be compared.

The value obtained when two similar quantities are compared by dividing one quantity with the other is called ratio**.**

Terms of a ratio

For a given ratio $a:b$ we say that a is the first term or antecedent and b is the second term or consequent. In the ratio $a:b$, a is the antecedent and b is the consequent.

Comparison of ratios

Two ratios $a:b$ and $c:d$ can be compared in the following way:

If $\frac{a}{b} > \frac{c}{d}$ then $a:b > c:d$.

Types of ratios

1. A ratio $a:b$, where $a > b$ is called a ratio of greater inequality

   For a ratio of greater inequality, if x be a positive quantity then

   $$\frac{a+x}{b+x} < \frac{a}{b}$$

2. A ratio $a:b$, where $a < b$ is called a ratio of lesser inequality

   For a ratio of lesser inequality, if x be a positive quantity then

   $$\frac{a+x}{b+x} > \frac{a}{b}$$

Values of the quantities

If two quantities are in the ratio $a:b$ then the first and second quantities will be $\frac{a}{a+b}$ times and $\frac{b}{a+b}$ times the sum of the two quantities respectively.

Proportion

The equality of two ratios is called proportion. If $a:b = c:d$, then a, b, c, and d are called to be in proportion and the same can be represented as $a:b :: c:d$, and is read as " a is to b as c is to d."

Properties

I. a, b, c and d are respectively known as the first, second, third and fourth proportional.

II. The first and forth terms are called extremes and the second and third terms are called means.

III. Product of extremes= Product of means

IV.

- ✓ Invertendo

  If a:b = c: d then b: a= d:c

- ✓ Alternendo

  If a:b =c:d then a: c= b: d

- ✓ Componendo

  If a:b = c: d then (a+b): b = (c+d): d

- ✓ Dividendo

  If a:b = c: d then (a - b): b = (c - d): d

- ✓ Componendo and Dividendo

  If a:b = c: d then (a+b): (a-b) = (c+d): (c- d)

V. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ and $l, m, and\ n$ are any three non-zero numbers, then

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{la+mc+ne}{lb+md+nf}.$$

Continued Proportion

Three quantities a, b, and c are said to be in continued proportion if a:b :: b: c.

Here c is called the third proportional of a and b.

The mean proportional of a and c is $\sqrt{ac}$.

Variation

The change in variable parameters is called Variation.

Types of variation

a. Direct Variation

An increase(or decrease) in the first quantity if results in a corresponding proportionate increase(or decrease) in the second quantity, then the two quantities are said to vary directly with each other.

For example,

At constant speed, distance covered varies directly with time.

That is, distance covered $\propto$ time

b. Indirect Variation

An increase(or decrease) in the first quantity if it results in a corresponding proportionate decrease(or increase) in the second quantity, then the two quantities are said to vary indirectly with each other.

For example, speed of an object is in indirect variation with time.

EXAMPLES

Example 1. The salaries of Ram and Shyam are in the ratio of 5: 9. The sum of their salaries is ₹3500. Find their individual salaries.

Sol. Ram's salary = 5/14th of their total salary and

Shyam's salary = 9/14th of their total salary.

∴ Ram's salary = $\frac{5}{14} \times 3500 =$ ₹ 1,250.

Shyam's salary = $\frac{9}{14} \times 3500 =$ ₹ 2,250.

Example 2. If a: b= 3:5, then find (2a+3b): (3a+5b).

Sol. a:b= 3:5 => $\frac{a}{b} = \frac{3}{5}$

Dividing the numerator and denominator by b, we get

$$= \frac{2\frac{a}{b}+3}{3\frac{a}{b}+5}$$

Now putting the value of a/b = 3/5, we get

$$= \frac{2\frac{3}{5}+3}{3\frac{3}{5}+5} = \frac{21}{34}$$

Example 3. ₹ 784 was divided among A, B, C and D in such a manner that 4 times the share of A, three times the share of B , two times the share of C are each equal to twelve times the share of D. Find their shares.

Sol. let the share of A, B, C, and D be a, b, c, and d respectively. Then,

4a= 3b= 2c = 12d ….(1)

a+b+c+d = 784 ….(2)

expressing a, b, and c in terms of d, we get a= 3d, b= 4d, c= 6d, now substituting the values in eq(2), we get

3d+ 4d+ 6d+d = 784

∴ d= 56.

So the share of A, B, and C are ₹168, ₹224, ₹336 respectively.

Example 4. If the incomes of P and Q are in the ratio of 3: 4 and their expenditures in the ratio 4: 5. Find the ratio of their savings, given that Q saves a third of his income.

Sol.

Let income of P and Q be $3x$ and $4x$, and

Expenditure $4y$ and $5y$ respectively

So, saving of P= $3x-4y$

and saving of Q= $4x-5y$

also, given

$4x-5y = 1/3(4x)$

Or $8x = 15y$

Let $x = 15k$ $y = 8k$

$$\frac{3x-4y}{4x-5y} = \frac{3(15k)-4(8k)}{4(15k)-5(8k)} = \frac{13k}{20k} = \frac{13}{20}$$

So, ratio of their savings is 13 : 20.

Example 5. The expenses per month of John' s car are partly constant and partly vary with the number of kilometers he travels in that month. When he travels 100 km in a month, the total expenses come to ₹3200. If he travels 150km, it is ₹3800. Find total expenses, if he travels 250 km in a month.

Sol.

Let total expenses be T, F be the fixed part and V be the variable part

So, T= F + V

The variable part varies with number of km travelled in the month.

⇨ V= pn ; where n is the number of *km* travelled in a month, and *p* is constant of proportionality

∴ T= F + pn

From the given data, we get

3200= F+ p(100) ---(1)

3800= F+ p(150)---(2)

Solving (1) and (2), we get

P= 12 and F = 2000

∴ If John travels 250 km, total expenses are

T = 2000 + 3000= ₹ 5000.

EXERCISE

1. The number of 1 rupees, 50 paise and 25 paise coins are in the ratio $1:2:4$. If the total value of the coins is rupees 75, the number of 50 paise coins is

(a) 20 (b) 30 (c) 50 (d) 80

2. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ and its perimeter is 169 cm. The difference between the length of the longest and shortest sides is

(a) 13 cm (b) 26 cm (c) 39 cm (d) 52 cm

3. Three containers have their volumes in the ratio $3:4:5$. They are full of mixtures of milk and water. The first mixture contains milk and water in the ratio $4:1$, the second in the ratio $3:1$ and the third in the ratio $5:2$ respectively. The contents of all these three mixture are poured into a fourth container. The ratio of milk and water in the fourth container is

(a) $141:37$ (b) $123:31$ (c) $157:53$ (d) $7:3$

4. If 10% of $x = 20\%$ of $y = 30\%$ of z, then the ratio of x, y and z is

(a) $1:2:3$ (b) $2:3:4$ (c) $3:4:5$ (d) $1:3:4$

5. In 60 litres mixture of milk and water the ratio of milk and water is $3:1$. How much water should be added in the mixture to make the ratio $2:1$?

(a) 4.5 litres (b) 7.5 litres (c) 10.5 litres (d) 12.5 litres

6. The ratio of the amounts with A and B is 3: 4. If B gives ₹5 to A, then the ratio of the amounts with A and B is 4:3. If A gives ₹5 to B, then find the ratio of the amounts with them.

a. 3:5 b. 2:5 c. 4:5 d. 1:5

7. In a camp there were 700 men and had provision for 30 days at the rate of 2 ½ kg per day per man. If 100 men had left the camp, then the provisions would have lasted for x days at the rate of 3 ½ kg per day per man. Find the value of x.

a. 18 b. 20 c. 25 d. 15

ANSWERS

Ans. 1.(c) 2. (c) 3. (c) 4.(a) 5.(b) 6.(b) 7.(c)

NUMBERS - I

Numbers is one of the most important topics asked in competitive examinations.

Real numbers are broadly classified into rational and irrational numbers.

I.Rational numbers.

A number which can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is called a rational number.

For example,

5 is a rational number since it can be expressed as $\frac{5}{1}$ where 5 and 1 are integers and $1 \neq 0$. Similarly $\frac{3}{4}$ and $-\frac{5}{7}$ are rational numbers.

Recurring decimals

Recurring decimals are also rationals because it is possible to express them in terms of $\frac{p}{q}$.

Note. Between any two numbers there can be infinite number of rational numbers.

II.Irrational numbers

Numbers which are not rational are irrational numbers.

For example,

$\sqrt{3}, \sqrt{2}, \sqrt[2]{8}, \ldots.$

Note. π and **e** are irrational numbers.

Decimals:

(a) Any recurring or terminating decimal are rational numbers.

(b) Any non-recurring or non-terminating decimal are irrational numbers.

Integers

All integers are rational numbers. They are classified into negative, zero, and positive integers.

Positive integers are further classified as Prime Numbers and Composite Numbers.

I.Prime numbers

A number which doesn't have any factor apart from 1 and itself is called a prime number.

Examples,

$2, 3.5, 7, 41, 67, 103, \ldots$ are prime numbers.

There is no fixed formula to generate a prime number.

II.Composite numbers

Any number other than 1 and which is not a prime number is a composite number.

Examples,

$4, 6, 9, 21, \text{etc}$. are composite numbers.

Note. The number 1 is neither prime nor composite. The only prime even number is 2.

Relative primes

Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1.

For example,

The numbers 16 and 25 are co-prime because there is no common factor between them except 1.

Note. It is not necessary for the individual numbers to be prime.

Even and odd numbers

A number divisible by 2 is even whereas a number not divisible by 2 is odd.

- Sum of any two even numbers is always even
- Sum of an even and an odd number is always odd
- Sum of two odd numbers is always even.
- The product of any number of odd numbers is always odd
- The product of any number of numbers where there is at least one even number is even.

Perfect numbers

A number is called perfect number when sum of all its factors excluding the number itself is equal to the number itself.

For example,

$6, 28, 496, \ldots$

As factors of 6 are $1, 2, 3$ and their sum $= 1 + 2 + 3 = 6$.

Factors of 28 are $1, 2, 4, 7, 14$ and their sum $= 1 + 2 + 4 + 7 + 14 = 28$

Rules for divisibility

I. Divisibility by 2

A number having its unit digit divisible by 2 is always divisible by 2.

II. Divisibility by 3

A number is divisible by 3 if the sum of its digit is a multiple of 3.

For example,

Take the number 9423, the sum of the digits is $9 + 4 + 2 + 3 = 18$ which is a multiple of 3. Hence the number 9423 is divisible by 3.

III. Divisibility by 4

A number is divisible by 4 if the last two digits of the number is divisible by 4.

For example,

73269836 is divisible by 4 because the last two digits 36 is divisible by 4.

Note. A number is divisible by 2^n ***if the last*** n***-digits of the number is divisible by*** 2^n**.**

3697523936 is divisible be $2^5(=32)$ because last 5-digits number 23936 is divisible by 32.

IV. Divisibility by 5

A number is divisible by 5 if its last digit is either 0 or 5.

For example, $23870, 76345$

V. Divisibility by 6

A number is divisible by 6 if it is divisible both by 2 and 3.

Example, $42, 96,$ etc.

VI. Divisibility by 7

If twice the unit digit when subtracted from the rest of number is divisible by 7 then that number is divisible by 7.

For example,

Let us take the number 574. If we subtract twice the unit digit number 4 from the rest of the number 57 we get

$57-2\times4=49$, since 49 is divisible by 7, hence 574 is divisible by 7.

Take another number 976

$97-2\times6=85$ which is not divisible by 7, hence 976 is not divisible by 7.

VII. Divisibility by 9

A number is divisible by 9 if the sum of its digit is a multiple of 9.

For example,

Take the number 9423, the sum of the digits is $9+4+2+3=18$ which is a multiple of 9. Hence 9423 is divisible by 9.

VIII. Divisibility by 11

A number is divisible by 11 if the difference between the sum of digits in odd places and the sum of digits in the even places is either zero or multiple of 11.

For example,

take the number 135795,

Sum of digits at odd places $=5+7+3=15$ and

at even places $=9+5+1=15$

Difference $=15-15=0$, hence the number 135795 is divisible by 11.

Number of Factors of a Number

Let N is a composite number such that $N = a^p.b^q.c^r \ldots.$ where a,b,c are prime factors of N and p,q,r are positive integers.

Then, the number of factors of N is

$$(p+1)(q+1)(r+1)\ldots.$$

$540 = 2^2 \times 3^3 \times 5^1$ hence 540 has $(2+1)(3+1)(1+1) = 24$ factors.

Note. The result of the total number of factors above include the factor 1 and the number itself.

Sum of all the Factors of a Number

Let a number $N = a^p \cdot b^q \cdot c^r \ldots.$ where a,b,c are prime factors of N and p,q,r are positive integers, then, the sum of all the factors of N (including 1 and the number itself) is

$$\left(\frac{a^{p+1}-1}{a-1}\right)\left(\frac{b^{q+1}-1}{b-1}\right)\left(\frac{c^{r+1}-1}{c-1}\right)\ldots.$$

For example, take the number 120.

$120 = 2^3 \times 3^1 \times 5^1$

Hence, the sum of all the factors

$$= \left(\frac{2^{3+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right)\left(\frac{5^{1+1}-1}{5-1}\right)$$

$= 15 \times 4 \times 6$

$= 360$.

Number of co-primes to N and less than N.

Let a number $N = a^p \cdot b^q \cdot c^r \ldots.$ where a,b,c are prime factors of N and p,q,r are positive integers. Then,

The number of co-primes to N and less than N are

$$\varphi(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots.$$

For example,

$48 = 2^4 \times 3^1$

So $\varphi(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 16$

Remainders

1. Every number N can be written in the form dq+r

That is, $N = dq + r$ where q is quotient and r remainder when N is divided by d.

Thus, a number when divided by 7 leaving a remainder of 5 can be written in the form $7q+5$.

✓ When a number N is divided by divisor d and leaves a remainder of r then the number (N-r) is exactly divisible by d.

That is, $d/(\text{N-r})$

✓ When a number N is divided by divisor d and leaves a remainder of r then

a. N-r is the largest multiple of d less than or equal to N.

For example, when 37 is divided by 5, the remainder is 2;

so $37-2, \text{i.e.}, 35$ is the largest multiple of 5 less than 37.

b. Smallest multiple of d which is greater than or equal to N is

$$N+(d-r).$$

For example,

When 59 is divided by 8, the remainder is 3; hence the smallest multiple of 8 which is greater than 59 is $59+(8-3)=64$.

2. Suppose a number N is divided by a number d leaving remainder r. Now if the Number N is split into two numbers N_1 and N_2 then,

$$\text{rem}(d/N) = \text{rem}(d/N_1) + \text{rem}(d/N_2) \text{ where } N = N_1 + N_2$$

For example,

$$\frac{27}{7} = \frac{16}{7} + \frac{11}{7}$$

$$\text{rem}\left(\frac{22}{7}\right) = 6, \text{rem}\left(\frac{16}{7}\right) = 2, \text{rem}\left(\frac{11}{7}\right) = 4$$

$$6 = 2 + 4.$$

3. Suppose a number N when divided by d leaves a remainder of r. Again suppose N is multiplied by a certain factor k and divided by d then the new remainder is kr. That is,

If $\text{rem}(d/N) = r$

$$\text{rem}(d/kN) = kr$$

for example,

when 13 is divided by 9, the remainder is 4. Now when the dividend 13 is multiplied by 2 we get 26 and when this number is divided by 9, the remainder is 8 which is the same as the original remainder multiplied by 2.

Least Common Multiple (LCM) **and Highest Common Factor** (HCF)

Least common multiple (LCM) of two or more numbers is the least number which is divisible by each of the numbers.

And Highest Common Factor is the largest factor of two or more given numbers.

Properties:

I. Let a and b be two numbers then

$$a \times b = lcm(a,b) \times hcf\,(a,b)$$

II. Suppose any number N which when divided by p, q, or r leaves a remainder s in each case, then

$$N = k\left(lcm(p,q,r)\right) + s \quad \text{where } k = 0,1,2,\ldots$$

III. A number when divided by p, q or r leaves remainders s, t, and u respectively such that $p-s = q-t = r-u = v\,(say)$ then that number is of the form

$$k\left(lcm(p,q,r)\right) - v \quad \text{Where } k \text{ is a constant.}$$

IV. The largest number with which the numbers p, q or r are divided giving remainders of s, t, and u respectively is

$$hcf\,\{(p-s),(q-t),(r-u)\}$$

V. The largest number which divides the numbers p, q, and r leaving behind the same remainder is

$$hcf\,\{(p-q),(p-r)\}$$

Largest Power of a Number in $N!$**.**

This involves finding the largest power of a number contained within the factorial of a number.

For example,

Find the largest power of 5 contained in 229!

The required number is the sum of the quotients after successive divisions.

5	229	quotient 1
5	45	quotient 2
5	9	quotient 3
	1	

Add all these quotients to get 45+9+1= 55

$\therefore 5^{55}$ is the highest power of 5 contained in 229!

Product of any two consecutive numbers is always even.

a. Two consecutive numbers can be written as either n and $n-1$ or n and $n+1$. Hence any number of the form $n(n-1)$ or $n(n+1)$ are always even.

b. Out of three consecutive integers, one of them is divisible by 3. And at least one of three is definitely even. Hence the product of three consecutive numbers is always divisible by 6.

Three consecutive numbers are of the form $n-1, n$ and $n+1$. Hence their product $(n-1)n(n+1)=n(n^2-1)$ is always divisible by 6.

Product of n consecutive numbers is always divisible by $n!$.

NUMBERS-II

The Last Digit of any Power

The Last digits of the powers of any number follow a cyclic pattern . They repeat after certain numbers of steps. For example,

Number	Last digit
3^1	3
3^2	9
3^3	7
3^4	1
3^5	3

Since last digit of 3^5 is the same as the last digit of 3^1. Hence last digit of $3^5, 3^6, 3^7$, and 3^8 will be the the same as those of $3^1, 3^2, 3^3$, and 3^4. Then, again, the last digit of 3^9 will be the same *as* 3^1 and so on. So in power of 3 the last digits repeat after every 4 steps. That is, last digits of $3^1, 3^5, 3^9$ will be the same.

Example

Find the last digit of 2^{58}.

Last digit of $2^1 = 2$

Last digit of $2^2 = 4$

Last digit of $2^3 = 8$

Last digit of $2^4 = 6$

Last digit of $2^5 = 2$

Thus, last digits repeats after 4 steps. So the last digit of 2^{56} will be the same as 2^4. Hence the last digit of 2^{58}, that is 2^{56+2} will be the same as $2^{56} \cdot 2^2$, ie., 4.

Last Digit of a SUM or PRODUCT

To find the last digit of the sum of two numbers each of which is a power of some integer.

For example,

Find out the last digit of $2^{58} + 3^{43}$.

We have already looked at how to find out the last digits of powers like 2^{58} & 3^{43}.

Last digit of $2^{58} = \left(2^4\right)^{14} \cdot 2^2 = 4$

Last digit of $3^{43} = \left(3^4\right)^{10} \cdot 3^3 = 7$

So, last digit of $2^{58} + 3^{43} = 4 + 7 = 11$ i.e 1 .

Finding the remainders involving powers of numbers

What is the remainder when 3^{45} is divided by 4 ?

Solution: (**pattern method**)

Remainder of $3^1/4=3$

Remainder of $3^2/4=9$

Remainder of $3^3/4=3$

We see remainder repeats after 2 - steps.

So, remainder of 3^{44} when divided by 4 is the same as 3^2 when divided by 4, since 44 is multiple of 4.

Hence remainder is

remainder of $3^{44}\times 3^1=1\times 3=3$

Remainder Theorem Method

Remainder theorem says that when a function $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$.

Let us take an example to have better understanding.

When a function $f(x)=x^{41}-50x+76$ is divided by $(x-1)$, the remainder is $f(1)$.

That is,

$f(1)=1^{41}-50.1+76$

$=77-50=27$

Example. Find the remainder of the division $2^{72}/7$.

Sol.

Since remainder is in terms of power of 2, let us express denominator in terms of 2 i.e., 2^3-1

expressing 2^{72} in terms of 2^3 i.e

$2^{72}=\left(2^3\right)^{24}$

So, problem reduces to find the remainder

when $\left(2^3\right)^{24}$ is divided by $\left(2^3-1\right)$.

Thus, remainder as per remainder theorem will be

$(1)^{24}=1$

Example. Find the remainder of the division $2^{75}/9$

$9=2^3+1=2^3-(-1)$

So, remainder where $\left(2^3\right)^{25}$ is divided by 9 is

$(-1)^{25} = -1$

In case of negative remainders, we add divisor to get the reminder, that is, remainder $= -1 + 9 = 8$.

Fermat Theorem

If p is a prime number and $\boldsymbol{a}$ is not a multiple of p then

$$p \text{ divides } \left(a^{p-1} - 1\right)$$

Example:

Take $p = 5$ and $a = 2$, then $3^4 - 1$ is divisible by 5

Rules related to $a^n + b^n / a^n - b^n$.

(i) Rules related to $a^n - b^n$.

a. It is always divisible by $(a - b)$

b. where n is even it is always divisible by $a + b$

c. When n is odd it is not divisible by $a + b$

(ii) Rules related to $a^n + b^n$

a. It is never divisible by $a - b$

b. Where n is odd, it is divisible by $a + b$.

c. Where n is even it is not divisible by $a + b$.

Example

$16^n + 1$ can be expressed as $(16)^n + (1)^n$

Hence,

(a) It is not divisible by $(16 - 1) = 15$.

(b) When $2^{75} / 9$ is odd as in $16^{73} + 1^{73}$, then it is divisible by $16 + 1 = 17$.

(c) When n is even as in $16^{72} + 1^{72}$, then it is not divisible by $16 + 1 = 17$.

MENSURATION

A figure lying in a plane is called plane or planar figure. Triangles, rectangles, and circles are some of the examples of plane figures.

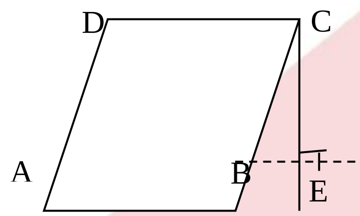
A plane figure is made of line segments or curves or both. The part of the plane whicjh is enclosed by simple closed figures is called plane region. The magnitude of a plane region is called its area. The perimeter of a closed planar figure is the total length of the lines bounding the figure.

I. Area of a rectangle = length × breadth

II. Area of a parallelogram = base × height

III. Area of parallelogram ABCD = (AB) (CE)

IV. Area of triangle



$\therefore$ The area of a triangle= $\frac{1}{2} \times base \times height$

The area of triangle is also given by $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b, c are the lengths of the sides of a triangle and s is its semi-parameter, i.e.,

$s = \frac{a+b+c}{2}$. This relation is known as Heron's formula.

Area of a Trapezium

$= \frac{1}{2}$(sum of the parallel sides)(distance between them).

Area of Rhombus

$= \frac{1}{2}$(product of the diagonals)

Area of a Square

= $(\text{side})^2$

$= \frac{1}{2}(diagonal)^2$

Area of a regular Polygon

$= \frac{1}{2}$(Perimeter of polygon)(Perpendicular distance from centre to any side)

Area of an Equilateral Triangle

$$= \frac{\sqrt{3}}{4} \times (side)^2$$

Area of an Isosceles Triangle

If two equal sides are of length *a* and the third side is of length *b*, then,

Area of the triangle= $\frac{b}{4}\sqrt{4a^2 - b^2}$

Area of a right triangle

= ½ ×a ×b where a is altitude and b is base length.

In- radius(r)

In- radius of a triangle is given by the formula,

$r = \frac{\Delta}{s}$ where Δ is the area of the triangle and s is its semi-perimeter

Circum-radius(R)

Circum-radius of a triangle is given by the formula

$$R = \frac{abc}{4\Delta}$$

Circles

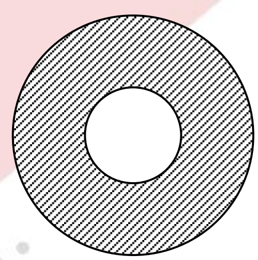
I. Circumference of a circle = π d = 2πr

II. Area of circle = πr^2

III. Area of a ring

The area enclosed between two concentric circles is known as a ring

In the fig. shaded region represents a ring. Let the radius of the smaller circle be r and the radius of the bigger circle be R

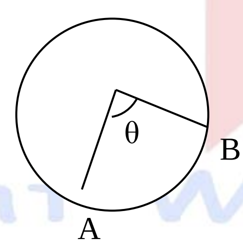


The area of the ring= $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$.

Sector of a Circle

The length of an arc of the the sector of a circle = $\frac{\theta}{360^0} \times 2\pi r$

The area of the sector of a circle= $\frac{\theta}{360^0} \times \pi r^2$



Solids

A solid is three dimensional object. In general, any object occupying space can be called a solid.

Some solids, like prisms, cubes, and cuboids have plane or flat surfaces, while some solids, like cone, cylinder have curved surfaces as well as flat surfaces. Lateral surface area of a solid having a curved surface is referred to as *curved surface area*.

Volume of solids

Prism

A prism is a solid in which two congruent and parallel polygons form the top and the bottom faces. The lateral faces are parallelograms.

The line joining the centres of the two parallel polygons is called the axis of the prism and the length of the axis is called height of the prism.

Right prism

If two parallel and congruent polygons are regular and, if the axis is perpendicular to the base, then the prism is called a right prism. The lateral surfaces of a right prism are rectangles.

Properties of Prisms:

1. The number of lateral faces = the number of sides of the base
2. The number of edges of a prism = the number of sides of the base × 3
3. The sum of the lengths of the edges = 2(perimeter of base)+ the number of sides × height
4. For a prism. Whose base is a polygon,
   Number of vertices + Number of faces = Number of edges + 2
   This is known as Euler's formula, that is, $V+F=E+2$
5. Lateral surface area of a prism
   = Perimeter of the base × height
6. Total surface area of a prism
   = lateral surface area + 2(area of base)
7. Volume of a prism
   = area of base × height

Cuboids

In a right prism if the base is a rectangle then it is called a cuboid. For example, a match box, a brick, a room etc.

Suppose the three dimensions of the cuboids, that is , its length= l, breadth= b *and* height = h *then*

I. Lateral surface area = $2(l+b)h$

II. Total surface area = $2(lb+bh+lh)$ sq. units.

III. Volume of a cuboid= lbh cubic units

IV. Diagonal of a cuboid $= \sqrt{l^2+b^2+h^2}$ units

Cube

In a cuboid, if all the dimensions, i.e., its length, breadth and height are equal, then it is called a cube. All the edges of a cube are equal in length. Thus, the size of a cube is completely determined by its edge.

If the edge of a cube be a units, then

I. The lateral surface area = 4 a^2

II. The total surface area = lateral surface area + 2(area of base) = $4a^2 + 2a^2 = 6a^2$

III. Volume of the cube = a^3

IV. The diagonal of the cube = $\sqrt{a^2+a^2+a^2} = \sqrt{3}a$

Right circular Cylinder

Let the base of the cylinder be of radius r and its height h then

I. Lateral surface area = $2\pi rh$

II. Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(r+h)$

III. Volume = area of base × height = $\pi r^2 h$

Pyramid

A pyramid is a solid obtained by joining the vertices of a polygon to a point in the space by straight lines. The polygon is the base of the Pyramid. Triangles are formed with each side of the base and the point in the space. The point in the space where all the triangles meet is called the *vertex* of the pyramid.

Right pyramid

If the line joining the vertex of the pyramid and the centre of the base is perpendicular to the base, then the pyramid is called a right pyramid.

For a right pyramid with base perimeter P , height = h and slant height = l :

I. Lateral surface area = ½(perimeter of base)(slant height)

II. Total surface area = lateral surface area + area of base

III. Volume of pyramid= 1/3 × area of base × height

Right Circular Cone

It is obtained by the revolution of a right- angled triangle about one of its two perpendicular sides.

So for a cone of radius r , height h, and slant height l :

I. Curved surface area= πrl sq. units

II. Total surface area = curved surface area + area of base = $\pi rl + \pi r^2 = \pi r(r+l)$ sq. units

III. Volume of a cone = $\frac{1}{3}\pi r^2 h$ cubic units

Sphere

I. Surface area of a spehere = $4\pi r^2$

II. Volume of a sphere = $\frac{4}{3}\pi r^3$

Hemisphere

I. Curved surface area of a hemisphere = $2\pi r^2$

II. Total surface area of a hemisphere= $3\pi r^2$

III. Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Spherical Shell

I. Thickness = $R-r$ where R = outer radius and r = inner radius

II. Volume = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

III. Total surface area of a hemispherical shell

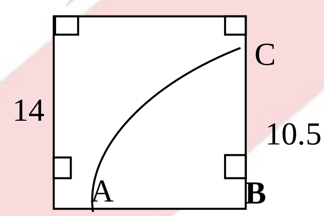
= curved surface area of outer hemisphere + curved surface area of inner hemisphere + area of ring

= $3\pi R^2 + \pi r^2$

EXAMPLES

Example 1. A goat is tied to one corner of a square plot of side 14 ft with a rope 10.5 ft long. Find the area the goat can graze and also the area it cannot graze.

Solution. Area of square = $\frac{90}{360}\times\frac{22}{7}\times 10.5\times 10.5 =$ sq. ft.



The area that goat can graze is sector ABC.

The radius of the sector is 10.5 ft, which is the length of the rope and angle of the sector is 90^{0}.

Area ABC = $\frac{90}{360}\times\frac{22}{7}\times 10.5\times 10.5 = 86.625$ sq. ft

Area that the goat cannot graze

= 196—86.625 = 109.375 sq. ft

Example 2. A circular path runs around and outside a circular garden of diameter 42m. if the difference between the outer circumference of the path and the circumference of the garden is 88m, find the width of the path,

Solution. circumference of inner circle = $2\times\frac{22}{7}\times 42 = 264\,m$

Circumference of the outer circle = 264 + 88= 352 m

$2\pi R = 352$, where R is the outer radius of the path

⇨ $R = \dfrac{352 \times 7}{2 \times 22} = 56\,cm$

⇨ Width of the path = Outer radius – Inner radius = 56 – 42 = 14 m

Example 3. A circle is placed in a rectangle such that it touches both the lengths of the rectangle. If the radius of the circle is one-fifth of the length of the rectangle, then find the ratio of the area of the region in the rectangle that is not covered by the circle to the area of the circle.

a. 21: 17 b. 17 : 11 c. 24: 11 d. 25: 11

Solution.

Let l be the length of the rectangle.

Radius (r) of the circle = $1/5$ (the length of the rectangle)

$= \dfrac{l}{5}$

∴ Breadth = 2 × radius of the circle

$= 2 \times \dfrac{l}{5} = \dfrac{2l}{5}$

Required ratio = area of the region in the rectangle that is not covered by the circle : Area of the circle = $l \times \dfrac{2l}{5} - \pi r^2 : \pi r^2$

$= \dfrac{2l^2}{5} - \dfrac{22}{7}\left(\dfrac{l}{5}\right)^2 : \dfrac{22}{7}\left(\dfrac{l}{5}\right)^2$

$= 24:11$

Example 4. If a regular hexagon is inscribed in a circle of radius 2 cm, then find rge area of polygon in cm^2

a. $6\sqrt{3}$ b. $12\sqrt{3}$ c. $4\sqrt{3}$ d. $24\sqrt{3}$

Solution.

Area of hexagon = 6 × Area of equilateral triangle = $6 \times \dfrac{\sqrt{3}}{4} \times 2^2 = 6\sqrt{3}$

Example 5. The dimensions of a room are $12\,ft \times 7\,ft \times 5\,ft$. Find

a. The diagonal of the room.

b. The cost of flooring at the rate of ₹ 3 per ft^2.

c. The cost of white- washing of the inside of the room excluding the floor at the rate of ₹ 4 per ft^2.

Solution.

a. The diagonal of the room = $\sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 7^2 + 5^2} = \sqrt{218}\,m$

b. In order to find the cost of flooring, we should find the area of the base.

Base area = $lb = 12 \times 7 = 84\,ft^2$

$\therefore$ Cost of flooring = $84 \times 3 =$ ₹ 252.

c. Total area which has to white washed = $2(lb+bh)+lb$

$$= 2(12+7)5 + 12\times 7 = 274\ ft^2$$

$\therefore$ Cost of white- washing = $274 \times 4 =$ ₹ 1096.

Example 6. The diameters of the top and bottom portions of a bucket are 56 cm and 14 cm, respectively. The height of the bucket is 72 cm. find the:

a. Area of metal sheet required to make the bucket. (without lid)

b. Volume of water which the bucket can hold.

Solution.

The bucket is in a cone frustum form with R = 28 cm, r = 7 cm and h = 72 cm.

a. Area of metal sheet required

= curved surface area + Area of bottom base = $\pi r(R+r)l + \pi r^2 h$

$$l = \sqrt{(R-r)^2 + h^2} = \sqrt{(28-7)^2 + 72^2} = 75cm$$

$\therefore$ Area of metal sheet = $\frac{22}{7} \times 75(28+7) + \frac{22}{7} \times 7^2 = 8404\,cm^2$

b. Amount of water which the bucket can hold

$$= \frac{1}{3}\pi\left(R^2 + Rr + r^2\right)h$$

$$= \frac{1}{3} \times \frac{22}{7}\left(28^2 + 28\times 7 + 7^2\right)72 = 77616\,cm^3$$

Example 7. A thin hollow hemispherical sailing vessel is made of metal surmounted by a conical canvas tent. The radius of the hemisphere is 14 ft and the total height of the vessel including the height of tent id 28ft. find the area of metal sheet and the canvas required.

Solution.

Total height , H = 28 ft

Radius of the hemisphere = r = 14 ft.

∴ Height of conical tent = h = H- r = 28- 14= 14 ft

Radius of the base of the cone = radius of the hemisphere = 14 ft

∴ Area of canvas required = $2\pi r^2 = 2\times\frac{22}{7}\times 14\times 14 = 1232\ ft^2$

$$= \frac{22}{7}\times 14\times\sqrt{14^2+14^2} = 44\times 14\sqrt{2} = 616\sqrt{2}\ ft^2$$

Area of metal sheet required = Surface area of hemisphere

$$= 2\pi r^2 = 2\times\frac{22}{7}\times 14\times 14 = 1232\ ft^2$$

EXERCISE

1. A cone and a hemisphere have equal bases and equal volumes. What is the ratio of their heights?

a. 1: 2 b. 3:1 c. 2:1 d. None of these

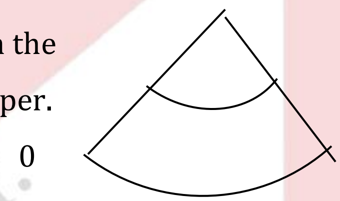
Ans.

2. The lengths of the diagonals of a rhombus are 9cm and 12 cm. find the distance between any two parallel sides of the rhombus.

a. 7.2 cm b. 8 cm c. 7.5 cm d. 6.9 cm

Ans

3. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, find the shaded area swept by the wiper.



a. 102. 66 cm b. 205.34 cm

b. 51.33 cm d. 208.16 cm

Ans

4. A wire is looped in the form of a circle of radius 35 cm, it is then bent into a square form. What will be the legth of the diagonal of the largest square possible?

a. 55 cm b. $55\sqrt{2}$ cm c. $110\sqrt{2}$ cm d. 110 cm

Ans

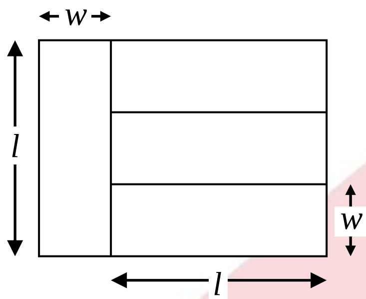
5. The radius of circle is so increased that its circumference increases by 20% . the area of the rhen increases by

a. 22% b. 44 % c. 24 % d. 35 %

6. The biggest possible cube is taken out of a right solid cylinder of radius 15 cm and height 20 cm respectively. What will be the volume of the cube?

a. 600 cm^3 b. 800 cm^3 c. 750 cm^3 d. 900 cm^3

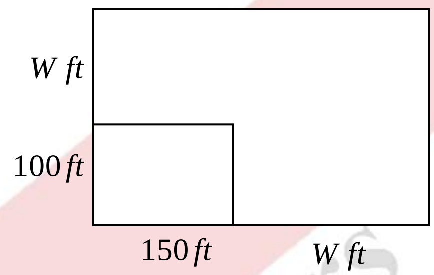
7. A painting consists of four differently coloured rectangles each of which has a length l and width w. If the area of the painting is 4800 square meters, what is the width (w) of each of the four rectangles (in meters)



(a) 15 (b) 20

(c) 25 (d) 30

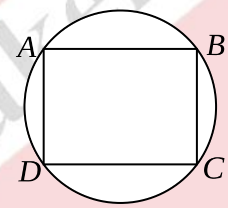
8. The smaller rectangle in the figure shown represents the original size of a parking lot before its length and width were each extended by W feet to make the larger rectangular lot shown. If the area of the enlarged lot is twice the area of the original lot, the value of W is



(a) 25 ft (b) 50 ft

(c) 75 ft (d) 50 ft

9. Square $ABCD$ is inscribed in a circle O. The area of the whole circular region is 64π. The area of the square region $ABCD$ is



(a) 64 (b) 128 (c) 256 (d) 512

10. In a triangle, the sum of any two sides exceed the third side by 6 cm. find its area(cm^2)

(a) $12\sqrt{3}$ (b) $9\sqrt{3}$ (c) $15\sqrt{3}$ (d) $18\sqrt{3}$

ANSWERS

Ans. 1. (C) 2. (a) 3. (a) 4. . (b) 5. (b) 6. (b) 7. (b) 8. (b) 9. (b) 10. (b)

GEOMETRY

Angle

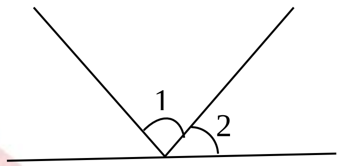
I. Right

II. Obtuse

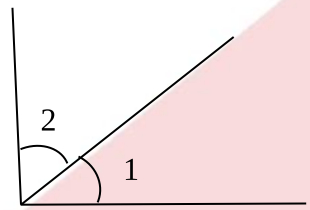
III. Straight

IV. Reflex

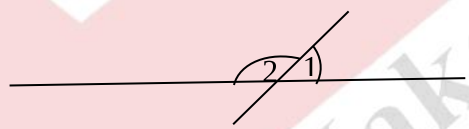
V. Adjacent



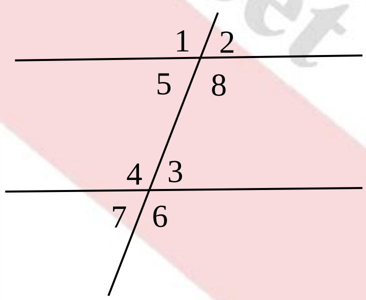
VI. Complementary



VII. Supplementary



VIII. Vertical



3, 4, 5, 8 – interior angles
1, 2, 6, 7 – exterior angles

Alternate: interior – $\angle 3 = \angle 5$

exterior -- $\angle 2 = \angle 7$ & $\angle 1 = \angle 6$

TRIANGLE

Triangle inequality

1. The sum of any two sides of a triangle is greater than third side.
2. The difference of any two sides is smaller than the third side.

Sine rule

If a, b, c are the three sides of a triangle, and A, B, and C are the angles in the triangles with A opposite a, B opposite b and C opposite c, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \text{ CosA}$$

Exterior angle = Sum of two interior angles

Area of a triangle

$\Delta = \frac{1}{2} \times base \times height$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = rs$

Where a, b, c are the sides of the triangle and s is semi-perimeter and r is the in-radius of the inscribed circle

$\Delta = \frac{1}{2} ac\sin B = \frac{1}{2} ab\sin C = \frac{1}{2} bc\sin A$

$\Delta = \frac{abc}{4R}$ where R is the radius of the circle circumscribing the triangle.

In a triangle

I. Ratio of medians = ratio of heights = ratio of R = ratio of r = ratio of angle bisector.

II. Ratio of areas = ratio of squares of corresponding sides.

Equilateral triangle

I. The in-centre and circum-centre lies at a point that divides the height in the ratio 2:1.

II. R= 2r always

III. Perimeter given constant, equilateral triangle has maximum area.

IV. Equilateral triangle inside a circle has maximum area.

V. Area of an equilateral triangle= $\frac{\sqrt{3}}{4} \times (\text{side})^2$

Isosceles triangle

Area of an isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$

Where b is the length of base and a is the length of equal sides.

Right triangle

Circum-radius(R) = hypotenuse/ 2

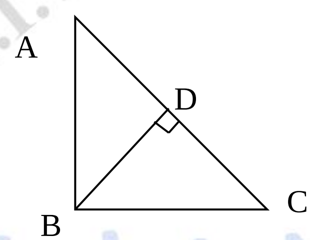
$\Delta = rs$

$r = \frac{bc}{a+b+c}$

$BC^2 = CD.AC$

$AB^2 = AD.AC$

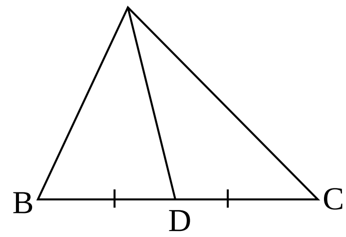
$BD^2 = AD.DC$



Medians

- A Median of a triangle divides the triangle in equal areas.
- The point at which medians meet is called centroid.

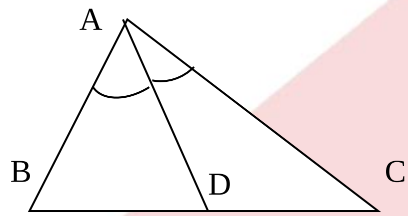
$$AB^2 + AC^2 = 2\left(AD^2 + DC^2\right)$$



Angle Bisector theorem

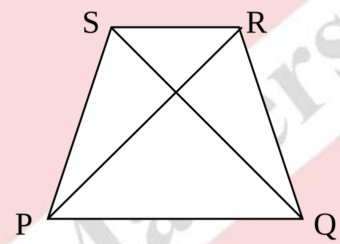
If AD be the angle bisector of the angle ∠BAC, then

$$\frac{BD}{AD} = \frac{AB}{AC}$$

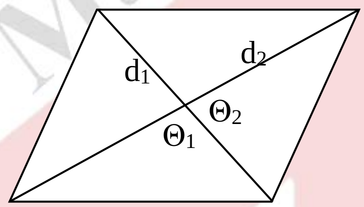


If two triangles have same base and lie between the same parallel lines then areas of the triangles will be equal

$$ar\left(\Delta PQR\right) = ar\left(\Delta PQS\right)$$



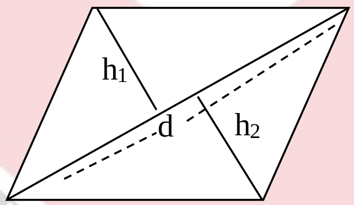
Quadrilateral



$$area = \frac{1}{2} \times d_1 \times d_2 \times \sin\theta_1$$

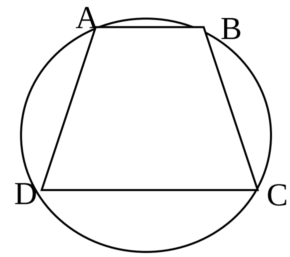
$$= \frac{1}{2} \times d_1 \times d_2 \times \sin\theta_2$$

$$area = \frac{1}{2} \times d \times \left(h_1 + h_2\right)$$



Property of cyclic quadrilateral :

$$AD \times BC + AB \times DC = AC \times BD$$



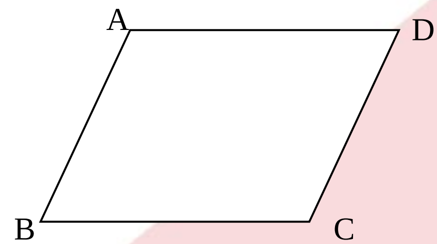
Parallelogram

Area = base × height

Perimeter = $= 2(a+b)$

Properties :

I. Diagonals bisect



II. Bisectors of angles of parallelogram form a rectangle

III. Parallelogram inscribed in a circle is rectangle

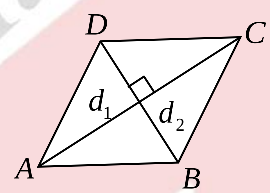
IV. Parallelogram circumscribed about a circle is rhombus

V. Opposite angles are equal

VI. $AC^2 + BD^2 = 2\left(AB^2 + BC^2\right)$

Rhombus

I. A parallelogram having all sides equal is called a rhombus



II. In a rhombus the diagonals are perpendicular to each other

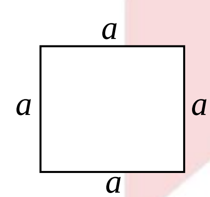
III. Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

IV. Each side of the rhombus $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

V. Diagonals of rhombus bisect each other at righ angles.

Square

(i) A square is a rectangle with a pair of adjacent sides equal



(ii) Each of the angles of a square is a right angle and each of the four sides is of the same length

(iii) Perimeter $= 4a$

(iv) Diagonal=$\sqrt{2}a$

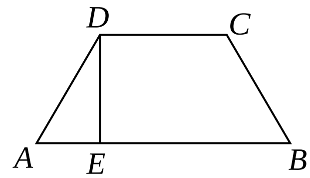
(v) Area $= a^2$

(vi) The diagonals of a square are perpendicular and of equal length.

Trapezium

(i) A quadrilateral having exactly one pair of parallel sides is a trapezium.

(ii) Area of a trapezium



$$=\frac{1}{2}\left(\text{The sum of parallel sides }\times\text{Perpendicular distance between parallel sides}\right)$$

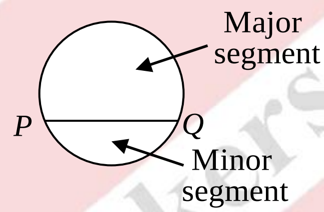
$$=\frac{1}{2}\times DE\times\left(AB+CD\right)$$

B

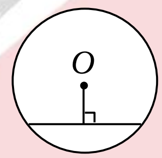
Circles

I. A line segment joining any two points on a circle is called the chord of the circle.

II. Let PQ be a chord of the circle. Then PQ divides the region enclosed by the circle into two parts. Each of these parts is called a segment of the circle. The segment containing the minor arc is called the minor segment and the segment containing the major arc is called the major segment

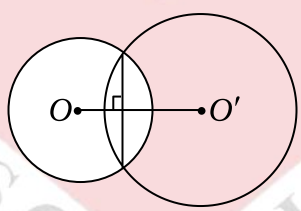


III. The perpendicular from the centre of a circle to a chord bisects the chord. Conversely, the line joining the centre of the circle to the mid points of a chord is perpendicular to the chord.

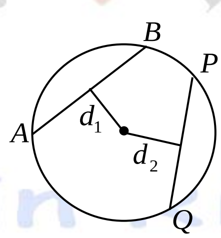


IV. There is one and only one circle passing through three non-collinear points.

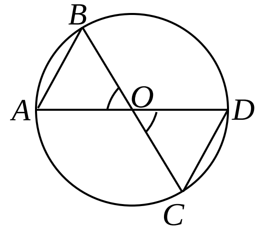
V. If two circles intersect in two points the line through the centers is perpendicular to the common chord



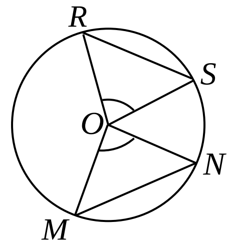
VI. Equal chords of a circle are equidistant from the centre. In the given figure suppose $AB=PQ$, then $d_1=d_2$



VII. Equal chords of a circle subtend equal angles at the centre. Conversely, if the angles subtended by two chords of a circle at the centre are equal, the chords are equal.



(i)

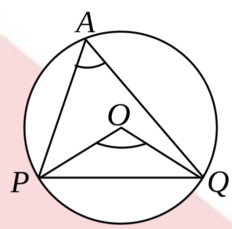


(ii)

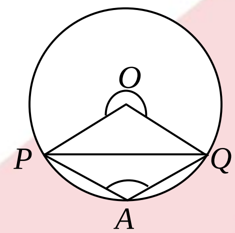
VIII. The angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.

In figure (i) $\angle POQ = 2\angle PAQ$

In figure (ii) $\angle POQ = 2\angle PAQ$

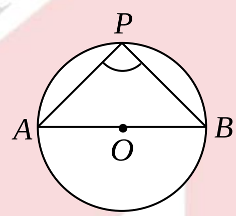


(i)



(ii)

IX. The angle in a semicircle is a right angle conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

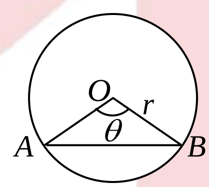


X. Circumference of a circle $= 2\pi \times \text{radius} = 2\pi r$

XI. Area of a circle $= \pi r^2$

XII. If θ (in degree) is the angle subtended by an arc AB at the centre, then

$$\text{arc } AB = \frac{2\pi r}{360^0} \times \theta$$



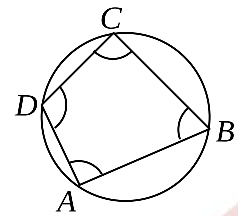
XIII. Area of sector $AOB = \frac{\pi r^2}{360^0} \times \theta = \frac{1}{2} \times \frac{2\pi r\theta}{360^0} \times r = \frac{1}{2} \times \text{arc } AB \times r$

Cyclic quadrilateral

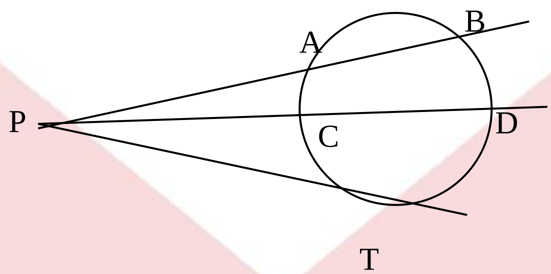
(i) A quadrilateral is called a cyclic quadrilateral if all its vertices lie on a circle. In the figure shown $ABCD$ is a cyclic quadrilateral.

(ii) The sum of the either pair of opposite angles of a cyclic quadrilateral is 180^0. In the figure shown

$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$



✓ If from a point P three lines are drawn such that PT is tangent and two other lines intersect the circles then



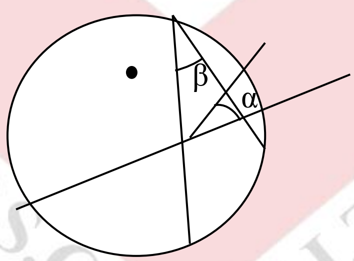
$PA.PB = PC.\ PD$

$PT^2 = PA.PB$

Alternate segment theorem

The angle between a tangent and a chord through the point of contact of the tangent is eual to the angle made by the chord in the alternate segment

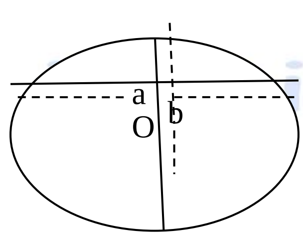
In the given figure α and β are alternate angles so they are equal.



Ellipse

Area : π a b

Perimeter : π(a+b)



Polygon

- Sum of interior- angles of convex polygon = $(2n-4)\pi/2$
- In a regular polygon of n sides

  a. Each interior angle = $\frac{2n-4}{n} \times \frac{\pi}{2}$

  b. Each exterior angle = $\frac{360^0}{n}$

EXERCISE

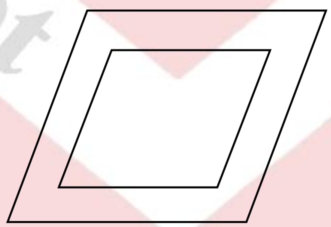
1. If ABC is a quarter circle and a circle is inscribed in it and if AB= 1 cm, find the radius of smaller circle:

$a. \sqrt{2}-1 \quad b. \left(\sqrt{2}+1\right)/2 \quad c. \sqrt{2}-\frac{1}{2} \quad d. 1-2\sqrt{2}$

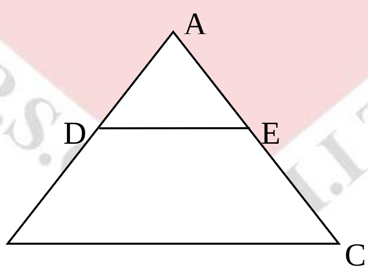
2. The parallel sides of a trapezium of area 176 sq. cm are 16 cm and 28 cm. Find the length of non-parallel sides, if they are of equal lengths.

$a. 10\ cm \quad b. 10\sqrt{2}\ cm \quad c. 10\sqrt{3}\ cm \quad d. 10\sqrt{6}\ cm$

3. Ram has a rhombus shaped farm-house ABCD. This farm is surrounded by a path of width 2ft, as shown in fig. If $\angle ADC = 30^0,\ AD = 10\,ft$. Then the area of the path is

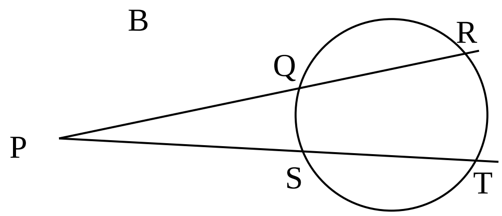


4. In a ΔABC, DE ∥ BC, then find the value of BC given AB= 8 cm, DE= 3 cm and BD= 6 cm



a. 9 cm   b. 4 cm   c. 12 cm   d. 16 cm

5. What is the length of PS given that PQ= 6cm, QR = 3cm, PT = 12 cm

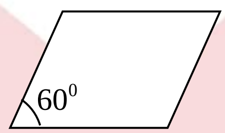


a. 3.5 b. 5.5 c. 6.5 d. 4.5

6. In a rhombus PQRS, PR= 32 cm and Qs= 24 cm. find the perimeter of the rhombus:
a. 80 cm b. 60 cm c. 112 cm d. 40 cm

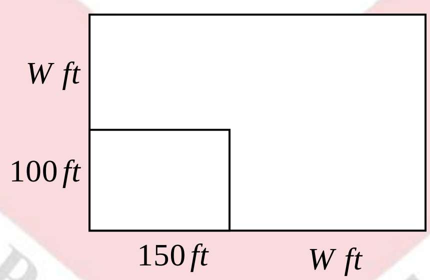
7. The sum of the interior angle of a polygon is 1440^0. How many sides does polygon have?
a. 10 b. 12 c. 5 d. none

8. The parallelogram shown has four sides of equal length. What is the ratio of the length of the shorter diagonal to the longer diagonal?



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

9. The smaller rectangle in the figure shown represents the original size of a parking lot before its length and width were each extended by W feet to make the larger rectangular lot shown. If the area of the enlarged lot is twice the area of the original lot, the value of W is



(a) 25 ft (b) 50 ft (c) 75 ft (d) 50 ft

ANSWER

Ans. 1.() 2. () 3. 112 ft² 4. (c) 5. (d) 6. (a) 7. (a) 8. (d) 9.(b)

How to Prepare for UPSC (logically with Mathematics Optional)

Mindset Makers for UPSC (Prepare in Right Way)

As an UPSC aspirant with **Science optional (For example Mathematics)**, We need completely a different approach. The interesting part is that if you have clear vision/mindset with such optional then you have high probability to secure the top rank. So is there any sigma rule for such strategy! The answer is – Yes in some sense. You must keep yourself isolated from the thought process where people have negative views about such optionals.

Yes it happens in this UPSC Preparation zone because there is a definite gap of mentors with equal interest in General Studies papers, Essay paper and such optionals. Mentors who are very good at GS and essay , they may not be good science optionals and who are teaching science optional they are experts of their domains mainly. Keeping all such obseravtions in mind , Mindset Makers tries to make easy this preparation with such optional combinations.

For this to happen, we are working in completely offline coaching module at-
YouTube (Upendra Singh : Mindset Makers for UPSC). A lecture Series with well planned course structure is going on . Here we teach Mathematics Optional and also do the mindset making for GS and Essay simultaneously with a scientific approach required for this optional combination. You must attend at least one lecture on youtube platform to feel the difference. Be Part of Disciplined Learning.

Mentor: Upendra Singh

Designation

Guide at Insights IAS and Nirman IAS (Prev.)
Sr. Faculty at Physics By Fiziks (Higher Studies(Mathematics) since year 2013)
Chairman: Patiyayat Farmer Producer Company Limited.
Associated in Policy making for Rural upliftment/education with Govt. Uttar Pradesh.

Educational Background

Alumnus, IIT Delhi (2011 Batch, AIR49)
UPSC CSE Journey- On the basis of one of top scores multiple times given by UPSC, I'm confident about each stage of CSE. Preliminary (Developed special strategies KNOWLEDGE+STRATEGY), Mains_ Good command over Essay and GS4+GS2, optional mathematics), Personality(Interview) - Developed it through experiences and learnings through reading books from multiple dimensions during UPSC journey, Part of Interview board for Higher studies.